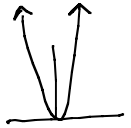


vertical stretch \Leftrightarrow horizontal compression

$$f(x) = 4x^2$$



$$f(x) = x^2$$



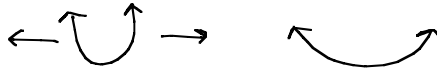
$$f(x) = \underline{4x^2}$$

vertical stretch

$$f(x) = \underline{x^2}$$

$$f(2x) = (2x)^2 = \underline{4x^2}$$

↑
replace x
with 2x

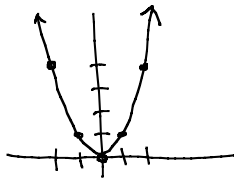


Section 2.4: Symmetry and Other Properties of Functions

Even vs. Odd functions

Even: $f(x) = x^2$

x	f(x)
-2	4
-1	1
0	0
1	1
2	4

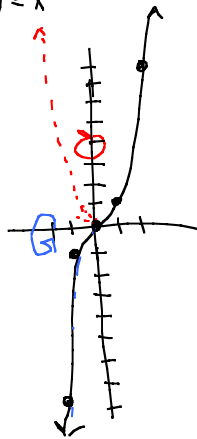


$$f(-x) = f(x)$$

symmetric about
the y-axis

Odd: $f(x) = x^3$

x	f(x)
-2	-8
-1	-1
0	0
1	1
2	8



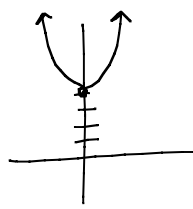
$$f(-2) = -f(2)$$

$$f(-1) = -f(1)$$

$$f(-x) = -f(x)$$

symmetry with
respect to the origin

example: Is $f(x) = x^2 + 4$ an even function or an odd function?

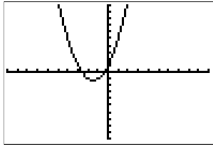


even - symmetric w.r.t. the y axis.

Is $f(-x) = f(x)$? yes - even

$$f(-x) = (-x)^2 + 4 = x^2 + 4 = f(x)$$

example: Is $f(x) = x^2 + 3x + 1$ even or odd?



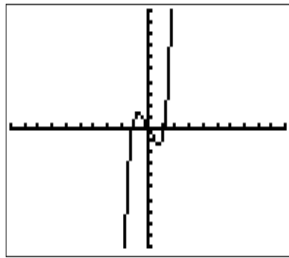
← neither even nor odd

Is $f(-x) = f(x)$?

$$\begin{aligned} f(-x) &= (-x)^2 + 3(-x) + 1 \\ &= x^2 - 3x + 1 \end{aligned}$$

not even

example: Is $f(x) = x^5 - 2x$ even or odd?



odd - symmetry w.r.t. the origin

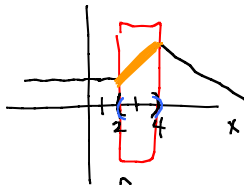
Does $f(-x) = -f(x)$?

$$f(-x) = (-x)^5 - 2(-x) = -x^5 + 2x = -f(x)$$

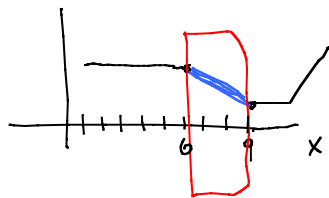
odd

Increasing and Decreasing Functions

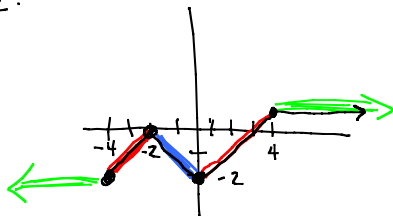
example: Draw a function whose graph is increasing on the interval (2, 4).



example: Draw a function whose graph is decreasing on the interval (6, 9)



example:



Over what interval is this function

a) Increasing?
(0, 4) and (-4, -2)

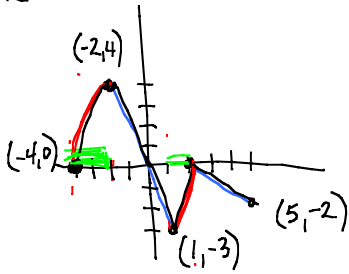
b) Decreasing?

$(-\infty, 0)$

c) Constant
 $(-\infty, -4)$ and $(4, \infty)$

example:

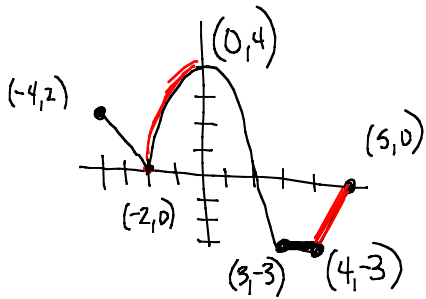
Pg 182



Increasing: $(-4, -2)$ and $(1, 2)$

Decreasing: $(-2, 1)$ and $(2, 5)$

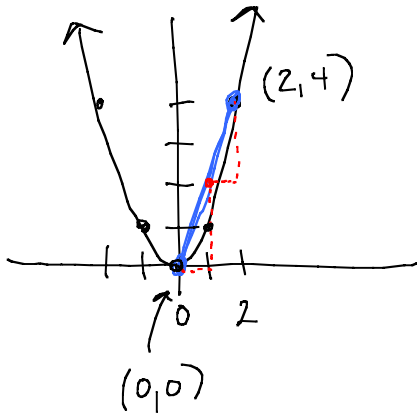
Constant: no where



Average Rate of Change

example: Find the average rate of change of $f(x) = x^2$

a) From 0 to 2



average rate of change = slope

$$m = \frac{4-0}{2-0} = \frac{4}{2} = 2$$

Average rate of change of a function f on an interval $[x_1, x_2]$ is

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$f(x) = x^2$$

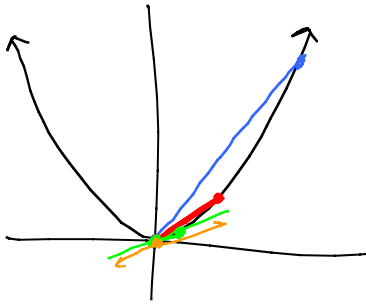
$$[0, 2]$$

$$x_1 = 0$$

$$x_2 = 2$$

$$f(x_1) = 0$$

$$f(x_2) = 4$$



$$\frac{4-0}{2-0} = \frac{4}{2} = 2$$

Section 2.4 : 1-7 (odd) 11-31 (all), 45-49