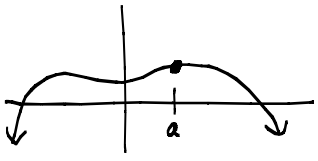
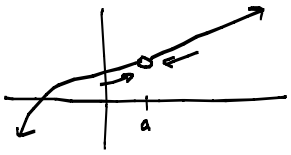
2.6 Continuity

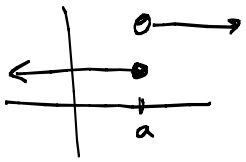
Informal def: A function is continuous if you don't pick your pen up off of the paper to draw its graph.



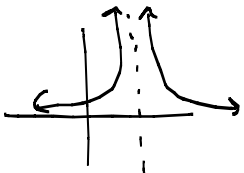
continuous



discontinuous



discontinuous


discontinuous

Formal definition for Continuity at a Point

A function will be continuous at $x = a$

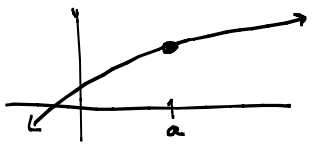
i. $f(a)$ is defined

ii. $\lim_{x \to a} f(x)$ exists

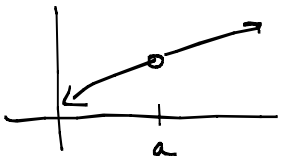
iii. $\lim_{x \to a} f(x) = f(a)$

} Continuity checklist

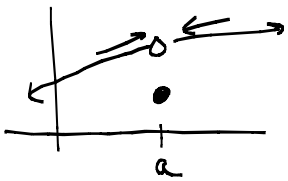
examples:



i. ✓
ii. ✓
iii. ✓



i. ✗
ii. ✓
iii. ✗



i. ✓
ii ✓
iii. ✗

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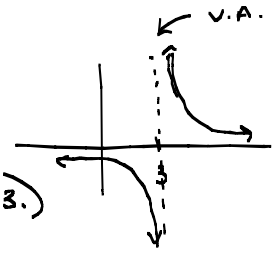
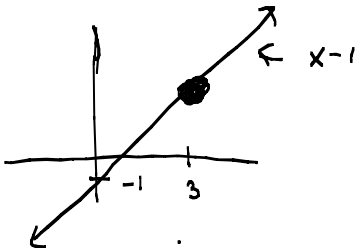
14. $g(x) = \dfrac{1}{x-3}$; $a = 3$



i. ✗ (The function is not defined at $x=3$.)

16. $f(x) = \begin{cases} \dfrac{x^2 - 4x + 3}{x - 3} & \text{if } x \neq 3 \\ 2 & \text{if } x = 3 \end{cases}$



i. $f(3) = 2$ ✓

ii. $\lim_{x \to 3} f(x) = \lim_{x \to 3} \dfrac{x^2 - 4x + 3}{x - 3} = \lim_{x \to 3} x - 1 = 2$ ✓

iii $\lim_{x \to 3} f(x) = f(3)$ ✓

Continuous at $a = 3$

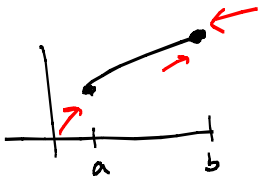
Continuity on an Interval

A function is continuous on an interval if it is continuous at every point in that interval. We need to consider the endpoints differently.

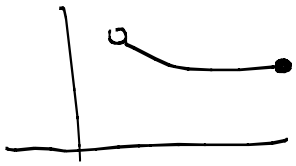
Definition: Continuity at an Endpoint

A function f is continuous from the left at b if $\lim_{x \to b^-} f(x) = f(b)$

A function f is continuous from the right at a if $\lim_{x \to a^+} f(x) = f(a)$



continuous on $[a,b]$

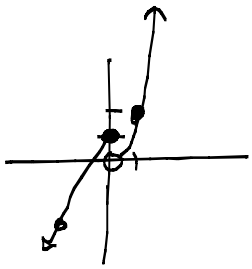


continuous on $(a,b]$

continuous on $[a,b)$

34. $f(x) = \begin{cases} x^3 + 4x + 1 & \text{if } x \leq 0 \\ 2x^3 & \text{if } x > 0 \end{cases}$

State the intervals of continuity.



$(-\infty, 0]$

$(0, \infty)$

Properties of continuity

Algebraic combinations of continuous functions are continuous

$f + g$

$f - g$

fg

$f/g \quad g \neq 0$

Types of continuous functions

① polynomial functions

② rational functions $\frac{p(x)}{q(x)}$ except where $q(x) = 0$

③ absolute value functions

④ trig functions — $\sin x$ and $\cos x$ are continuous everywhere

⑤ radical functions $f(x) = \sqrt[n]{x}$

⑥ composite functions

$f(x) = (x^2 + 3x + 1)^4$

Homework: Section 2.6: 9-27 (odd), 33-47 (odd), 55-59 (odd)