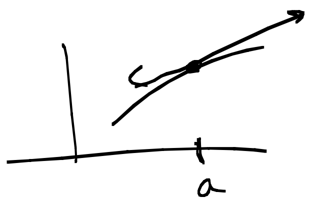
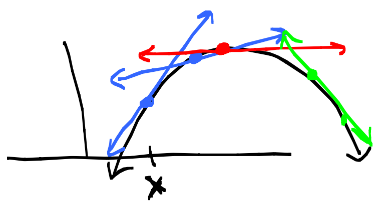
Section 3.1 Derivatives



The Derivative Function



f' : derivative function for f

$f'(x)$: the slope of the tangent line at a variable point $(x, f(x))$

"f prime of x"

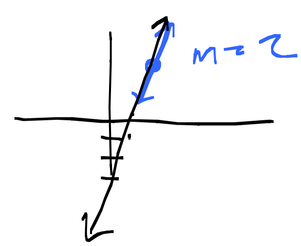
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

If $f'(x)$ exists, then f is differentiable at x.

example: Find the derivative function of $f(x) = 2x - 3$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{2x+2h-3-(2x-3)}{h} = \lim_{h \to 0} \frac{2h}{h} = 2$$

$$f(x+h) = 2(x+h) - 3 = 2x + 2h - 3$$
$$f(x) = 2x - 3$$



example: Find the derivative function of $f(x) = x^2 + 4$.

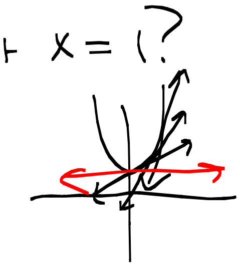
$$f(x+h) = (x+h)^2 + 4 \overset{\text{FOIL}}{=} (x+h)(x+h) + 4 = x^2 + 2xh + h^2 + 4$$
$$f(x) = x^2 + 4$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 4 - (x^2 + 4)}{h}$$

$$= \lim_{h\to 0} \frac{2xh+h^2}{h} = \lim_{h\to 0} 2x+h = 2x$$

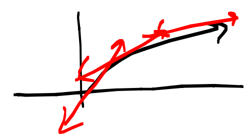
What is the slope of the tangent line at $x = 1$?

$$f'(1) = 2\cdot 1 = 2$$



Derivative Notation

$f'(x)$	f prime of x
f'	f prime
y'	y prime
$\frac{dy}{dx}$	dy dx
$\frac{d}{dx} f$	ddx of f



example: Let $f(x) = \sqrt{x}$. Compute $\frac{dy}{dx}$.

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(\sqrt{x+h}-\sqrt{x})}{h} \cdot \frac{(\sqrt{x+h}+\sqrt{x})}{\sqrt{x+h}+\sqrt{x}}$$

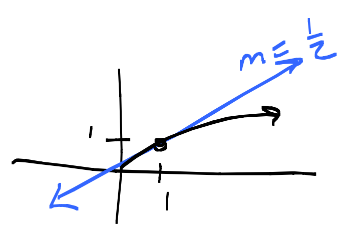
$$f(x+h) = \sqrt{x+h}$$
$$f(x) = \sqrt{x}$$

$$= \lim_{h\to 0} \frac{\cancel{x}+\cancel{h}-\cancel{x}}{\cancel{h}(\sqrt{x+h}+\sqrt{x})} =$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{x+h}+\sqrt{x}} = \frac{1}{\sqrt{x}+\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

What is the slope of the tangent line to f at $x = 1$?

$$f'(1) = \frac{1}{2\sqrt{1}} = \frac{1}{2}$$



Homework: Section 3.1: 23-35 (odd)