27. $f(x) = \frac{1}{\sqrt{x}}$; $a = \frac{1}{4}$ $\qquad \frac{1}{\sqrt{\frac{1}{4}}} = \frac{1}{\frac{1}{2}} = 2$

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \to 0} \frac{f\left(\frac{1}{4}+h\right) - f\left(\frac{1}{4}\right)}{h}$$

LCD $= \sqrt{\frac{1}{4}+h}$

$$= \lim_{h \to 0} \frac{\frac{1}{\sqrt{\frac{1}{4}+h}} - 2}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{\cancel{\sqrt{\frac{1}{4}+h}}} \cdot \cancel{\sqrt{\frac{1}{4}+h}} - 2 \cdot \sqrt{\frac{1}{4}+h}}{h\sqrt{\frac{1}{4}+h}}$$

mult. by the conjugate of the numerator

$$= \lim_{h \to 0} \frac{1 - 2\sqrt{\frac{1}{4}+h}}{h\sqrt{\frac{1}{4}+h}} \cdot \frac{1 + 2\sqrt{\frac{1}{4}+h}}{1 + 2\sqrt{\frac{1}{4}+h}}$$

$$= \lim_{h \to 0} \frac{1 - 4\left(\frac{1}{4}+h\right)}{\left(h\sqrt{\frac{1}{4}+h}\right)\left(1 + 2\sqrt{\frac{1}{4}+h}\right)}$$

$$= \lim_{h \to 0} \frac{\cancel{1} - \cancel{1} - 4\cancel{h}}{\left(\cancel{h}\sqrt{\frac{1}{4}+h}\right)\left(1 + 2\sqrt{\frac{1}{4}+h}\right)}$$

$$= \frac{-4}{\sqrt{\frac{1}{4}}\left(1 + 2\sqrt{\frac{1}{4}}\right)} = \frac{-4}{\left(\frac{1}{2}\right)(2)} = -4$$

35. $y = \frac{1}{t+1}$; $t = 1$

LCD $= 2(2+h)$

$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0} \frac{\frac{1}{2+h} - \frac{1}{2}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{\cancel{2+h}} \cdot 2\cancel{(2+h)} - \frac{1}{\cancel{2}} \cdot \cancel{2}(2+h)}{h \cdot 2(2+h)}$$

$$= \lim_{h \to 0} \frac{2 - (2+h)}{2h(2+h)}$$

$$= \lim_{h \to 0} \frac{-\cancel{h}}{2\cancel{h}(2+h)}$$

$$= \frac{-1}{4}$$

$$\underline{\underline{f'(x)}} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$f(x) = x^2 - 1$

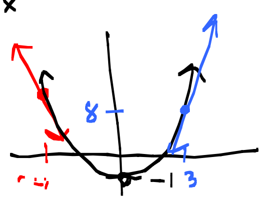
$f(x+h) = (x+h)^2 - 1$

$$= \lim_{h \to 0} \frac{(x+h)^2 - 1 - (x^2 - 1)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 \cancel{-1} \cancel{-x^2} \cancel{+1}}{h}$$

$$= \lim_{h \to 0} \frac{h(2x+h)}{h} = \lim_{h \to 0} 2x + h = \underline{\underline{2x}}$$

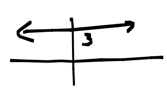
$f(x) = x^2 - 1$



3.2 Rules of Differentiation

① Derivative of the constant function

ex. $f(x) = 3$

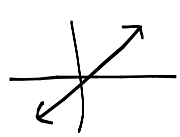


$f(x) = 7$

If c is a real number, then $\frac{d}{dx} c = 0$ (or $f'(x) = 0$)

② Derivative of a linear function

ex. Given $f(x) = mx + b$



If m and b are real numbers, then $\frac{d}{dx}(mx+b) = m$

③ Power Rule

If n is a positive or negative integer,

$$\frac{d}{dx} x^n = n x^{n-1}$$

ex. $\frac{d}{dx}x^2 = 2x^{2-1} \quad 2x$

$$\frac{d}{dx}x^3 = 3x^2$$

$$\frac{d}{dx}x^4 = 4x^3$$

④ Constant Multiple Rule

If c is a constant,

$$\frac{d}{dx}cf(x) = c\frac{d}{dx}f(x)$$

c gets pulled through

⑤ Sum and Difference Rules

$$\frac{d}{dx}\left(f(x)+g(x)\right) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

$$\frac{d}{dx}\left(f(x)-g(x)\right) = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$$

examples:

$$\frac{d}{dx}\left(3t^5\right) \underbrace{=}_{\text{constant multiple rule}} 3\frac{d}{dx}\left(t^5\right) \underbrace{=}_{\text{power rule}} 3\cdot 5t^4 = 15t^4$$

$$\frac{d}{dx}\left(x^2+1\right) \underbrace{=}_{\text{addition rule}} \frac{d}{dx}x^2 + \frac{d}{dx}1 \underbrace{=}_{\text{constant function}} 2x + 0 = 2x$$

$$\frac{d}{dx}\left(-10\sqrt{x}\right) \underbrace{=}_{\text{constant multiple rule}} -10\frac{d}{dx}x^{\frac{1}{2}} = -10\cdot\frac{1}{2}x^{-1/2} = -5x^{-1/2}$$

$\frac{1}{2}-1$

$$\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{d}{dx}\left(x^{-1}\right) = -1x^{-2} = \frac{-1}{x^2}$$

$-1-1$

Pg 118   8, 10, 12, 14, 16, 18, 20, 22, 24

8. $11t^{10}$

10. 0

12. $100v^{99}$

14. $10w^{11}$

✓16. $3t^{-1/2} = \frac{3}{\sqrt{t}}$

✓18. $\frac{s^{-1/2}}{8} = \frac{1}{8\sqrt{s}}$

20. $30x^4 - 1$

✓22. $3t^{-1/2} - 12t^2 = \frac{3}{\sqrt{t}} - 12t^2 = \frac{3\sqrt{t}}{t} - 12t^2$

24. $2t^{-1/2} - t^3 + 1 = \frac{2}{\sqrt{t}} - t^3 + 1 = \frac{2\sqrt{t}}{t} - t^3 + 1$

$$\frac{d}{dt} 6\sqrt{t} = \frac{d}{dt} 6 \cdot t^{1/2} = 6 \frac{d}{dt} t^{1/2} = 6 \cdot \frac{1}{2} t^{\frac{1}{2}-1} = 3t^{-1/2}$$

$$= \frac{3}{\sqrt{t}} = \frac{3\sqrt{t}}{t}$$

$$\frac{d}{ds} \frac{\sqrt{s}}{4}$$

22. $$\frac{d}{dt}\left(6\sqrt{t} - 4t^3 + 9\right) = \frac{d}{dt} 6\sqrt{t} - \frac{d}{dt} 4t^3 + \frac{d}{dt} 9$$

$$= 6 \cdot \frac{1}{2} t^{-1/2} - 12t^2 + 0$$

$$= 3t^{-1/2} - 12t^2$$

Homework: Section 3.2: 7-45 (odd)

Read about Slopes of Tangent Lines and Higher-Order Derivatives

18. $f(s) = \frac{\sqrt{s}}{4}$

$$\frac{d}{ds} \frac{\sqrt{s}}{4} = \frac{1}{4} \frac{d}{ds} \sqrt{s} = \frac{1}{4} s^{1/2} = \frac{1}{4} \cdot \frac{1}{2} s^{-1/2}$$