CALCULUS I

Practice Problems
Limits

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Table of Contents

Preface

Here are a set of practice problems for my Calculus I notes. If you are viewing the pdf version of this document (as opposed to viewing it on the web) this document contains only the problems themselves and no solutions are included in this document. Solutions can be found in a number of places on the site.

1. If you'd like a pdf document containing the solutions go to the note page for the section you'd like solutions for and select the download solutions link from there. Or,

2. Go to the download page for the site http://tutorial.math.lamar.edu/download.aspx and select the section you'd like solutions for and a link will be provided there.

3. If you'd like to view the solutions on the web or solutions to an individual problem you can go to the problem set web page, select the problem you want the solution for. At this point I do not provide pdf versions of individual solutions, but for a particular problem you can select "Printable View" from the "Solution Pane Options" to get a printable version.

Note that some sections will have more problems than others and some will have more or less of a variety of problems. Most sections should have a range of difficulty levels in the problems although this will vary from section to section.

I add problems to this document as I get a chance and so that means that sometimes some sections are apparently given more attention than others. Eventually I'll have a full complement of problems for all the sections, however it takes time to write the problems, write the solutions for each problem and format them for presentation on the web. If there aren't practice problems for a given section (or there aren't that many problems in a section) then know that I will eventually

get around to writing problems for the section (or more problems for the section as the case may be).

Please do not email me asking when I'm going to get problems for a particular section written. As I've already said, I write problems as I have time to do so. Please understand that I have my own classes to teach and other responsibilities to my department that I have to also take care of. This web site and all of its information has been written and maintained in my spare time and that often means that I don't have as much time to work on it as I'd like to.

Limits

Introduction

Here are a set of practice problems for the Limits chapter of my Calculus I notes. If you are viewing the pdf version of this document (as opposed to viewing it on the web) this document contains only the problems themselves and no solutions are included in this document. Solutions can be found in a number of places on the site.

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Here is a list of topics in this chapter that have practice problems written for them.

Limits At Infinity, Part II
Continuity
The Definition of the Limit – Problems for this section have not yet been written.

Rates of Change and Tangent Lines

1. For the function $f(x)=3(x+2)^2$ and the point *P* given by $x=-3$ answer each of the following questions.

(a) For the points *Q* given by the following values of *x* compute (accurate to at least 8 decimal places) the slope, m_{PQ}, of the secant line through points *P* and *Q*.

(i) -3.5	**(ii)** -3.1	**(iii)** -3.01	**(iv)** -3.001	**(v)** -3.0001
(vi) -2.5	**(vii)** -2.9	**(viii)** -2.99	**(ix)** -2.999	**(x)** -2.9999

(b) Use the information from **(a)** to estimate the slope of the tangent line to $f(x)$ at $x=-3$ and write down the equation of the tangent line.

2. For the function $g(x)=\sqrt{4x+8}$ and the point *P* given by $x=2$ answer each of the following questions.

(a) For the points *Q* given by the following values of *x* compute (accurate to at least 8 decimal places) the slope, m_{PQ}, of the secant line through points *P* and *Q*.

(i) 2.5	**(ii)** 2.1	**(iii)** 2.01	**(iv)** 2.001	**(v)** 2.0001
(vi) 1.5	**(vii)** 1.9	**(viii)** 1.99	**(ix)** 1.999	**(x)** 1.9999

(b) Use the information from **(a)** to estimate the slope of the tangent line to $g(x)$ at $x=2$ and write down the equation of the tangent line.

3. For the function $W(x)=\ln\left(1+x^4\right)$ and the point *P* given by $x=1$ answer each of the following questions.

(a) For the points *Q* given by the following values of *x* compute (accurate to at least 8 decimal places) the slope, m_{PQ}, of the secant line through points *P* and *Q*.

(i) 1.5	**(ii)** 1.1	**(iii)** 1.01	**(iv)** 1.001	**(v)** 1.0001
(vi) 0.5	**(vii)** 0.9	**(viii)** 0.99	**(ix)** 0.999	**(x)** 0.9999

(b) Use the information from **(a)** to estimate the slope of the tangent line to $W(x)$ at $x=1$ and write down the equation of the tangent line.

4. The volume of air in a balloon is given by $V(t)=\dfrac{6}{4t+1}$ answer each of the following questions.

(a) Compute (accurate to at least 8 decimal places) the average rate of change of the volume of air in the balloon between $t=0.25$ and the following values of *t*.

(i) 1	**(ii)** 0.5	**(iii)** 0.251	**(iv)** 0.2501	**(v)** 0.25001
(vi) 0	**(vii)** 0.1	**(viii)** 0.249	**(ix)** 0.2499	**(x)** 0.24999

(b) Use the information from **(a)** to estimate the instantaneous rate of change of the volume of air in the balloon at $t=0.25$.

5. The population (in hundreds) of fish in a pond is given by $P(t)=2t+\sin(2t-10)$ answer each of the following questions.

(a) Compute (accurate to at least 8 decimal places) the average rate of change of the population of fish between $t=5$ and the following values of *t*. Make sure your calculator is set to radians for the computations.

(i) 5.5	**(ii)** 5.1	**(iii)** 5.01	**(iv)** 5.001	**(v)** 5.0001
(vi) 4.5	**(vii)** 4.9	**(viii)** 4.99	**(ix)** 4.999	**(x)** 4.9999

(b) Use the information from **(a)** to estimate the instantaneous rate of change of the population of the fish at $t=5$.

6. The position of an object is given by $s(t)=\cos^2\left(\dfrac{3x-6}{2}\right)$ answer each of the following questions.

(a) Compute (accurate to at least 8 decimal places) the average velocity of the object between $t=2$ and the following values of *t*. Make sure your calculator is set to radians for the computations.

(i) 2.5	**(ii)** 2.1	**(iii)** 2.01	**(iv)** 2.001	**(v)** 2.0001
(vi) 1.5	**(vii)** 1.9	**(viii)** 1.99	**(ix)** 1.999	**(x)** 1.9999

(b) Use the information from **(a)** to estimate the instantaneous velocity of the object at $t=2$ and determine if the object is moving to the right (*i.e.* the instantaneous velocity is positive), moving to the left (*i.e.* the instantaneous velocity is negative), or not moving (*i.e.* the instantaneous velocity is zero).

7. The position of an object is given by $s(t)=(8-x)(x+6)^{\frac{3}{2}}$. Note that a negative position here simply means that the position is to the left of the "zero position" and is perfectly acceptable. Answer each of the following questions.

(a) Compute (accurate to at least 8 decimal places) the average velocity of the object between $t=10$ and the following values of *t*.

(i) 10.5	**(ii)** 10.1	**(iii)** 10.01	**(iv)** 10.001	**(v)** 10.0001
(vi) 9.5	**(vii)** 9.9	**(viii)** 9.99	**(ix)** 9.999	**(x)** 9.9999

(b) Use the information from **(a)** to estimate the instantaneous velocity of the object at $t=10$ and determine if the object is moving to the right (*i.e.* the instantaneous velocity is positive), moving to the left (*i.e.* the instantaneous velocity is negative), or not moving (*i.e.* the instantaneous velocity is zero).

The Limit

1. For the function $f(x)=\dfrac{8-x^3}{x^2-4}$ answer each of the following questions.

(a) Evaluate the function the following values of *x* compute (accurate to at least 8 decimal places).

(i) 2.5	**(ii)** 2.1	**(iii)** 2.01	**(iv)** 2.001	**(v)** 2.0001
(vi) 1.5	**(vii)** 1.9	**(viii)** 1.99	**(ix)** 1.999	**(x)** 1.9999

(b) Use the information from **(a)** to estimate the value of $\lim\limits_{x\to 2}\dfrac{8-x^3}{x^2-4}$.

2. For the function $R(t)=\dfrac{2-\sqrt{t^2+3}}{t+1}$ answer each of the following questions.

(a) Evaluate the function the following values of *t* compute (accurate to at least 8 decimal places).

(i) -0.5	**(ii)** -0.9	**(iii)** -0.99	**(iv)** -0.999	**(v)** -0.9999
(vi) -1.5	**(vii)** -1.1	**(viii)** -1.01	**(ix)** -1.001	**(x)** -1.0001

(b) Use the information from **(a)** to estimate the value of $\lim\limits_{t\to -1}\dfrac{2-\sqrt{t^2+3}}{t+1}$.

3. For the function $g(\theta)=\dfrac{\sin(7\theta)}{\theta}$ answer each of the following questions.

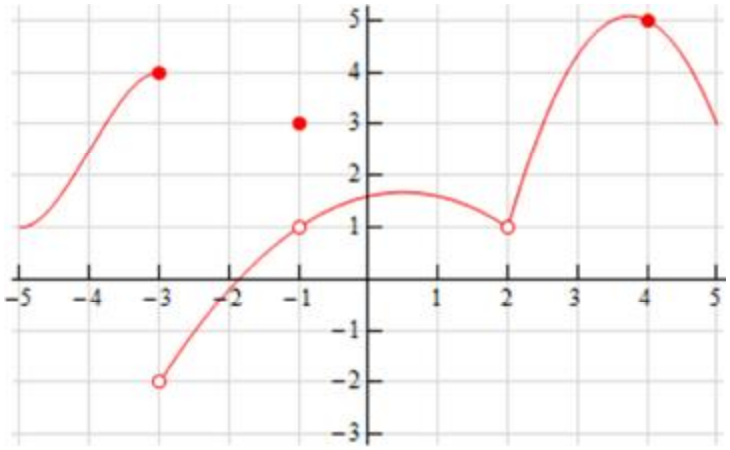
(a) Evaluate the function the following values of θ compute (accurate to at least 8 decimal places). Make sure your calculator is set to radians for the computations.

(i) 0.5 **(ii)** 0.1 **(iii)** 0.01 **(iv)** 0.001 **(v)** 0.0001
(vi) -0.5 **(vii)** -0.1 **(viii)** -0.01 **(ix)** -0.001 **(x)** -0.0001

(b) Use the information from **(a)** to estimate the value of $\lim\limits_{\theta\to 0}\dfrac{\sin(7\theta)}{\theta}$.

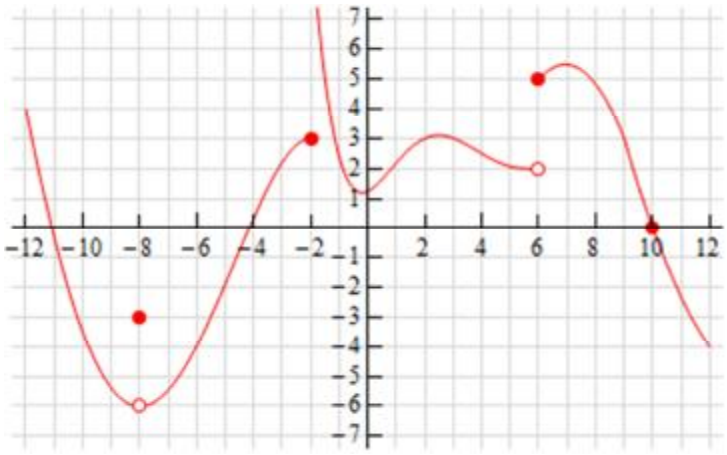
4. Below is the graph of $f(x)$. For each of the given points determine the value of $f(a)$ and $\lim\limits_{x\to a} f(x)$. If any of the quantities do not exist clearly explain why.

(a) $a=-3$ **(b)** $a=-1$ **(c)** $a=2$ **(d)** $a=4$



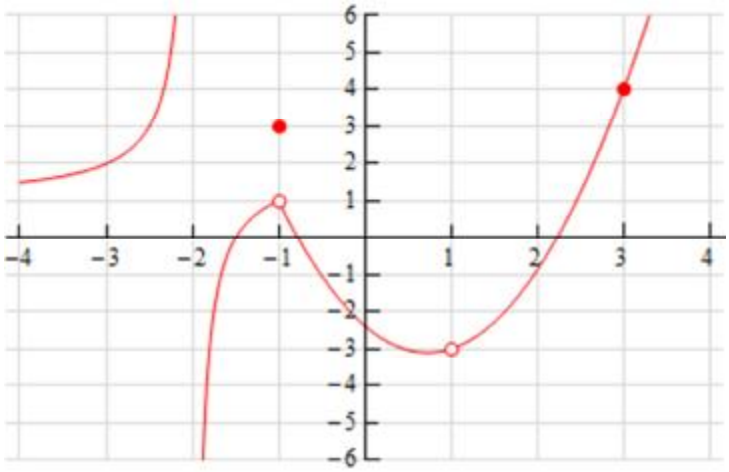
5. Below is the graph of $f(x)$. For each of the given points determine the value of $f(a)$ and $\lim\limits_{x\to a} f(x)$. If any of the quantities do not exist clearly explain why.

(a) $a=-8$ **(b)** $a=-2$ **(c)** $a=6$ **(d)** $a=10$



6. Below is the graph of $f(x)$. For each of the given points determine the value of $f(a)$ and $\lim_{x\to a} f(x)$. If any of the quantities do not exist clearly explain why.

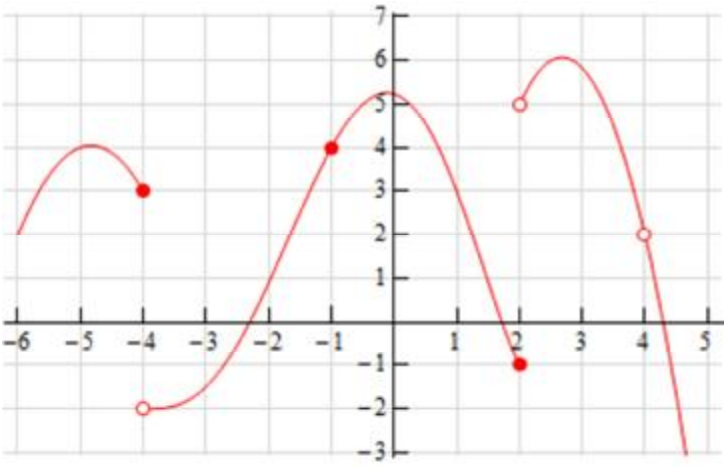
(a) $a=-2$ **(b)** $a=-1$ **(c)** $a=1$ **(d)** $a=3$



One-Sided Limits

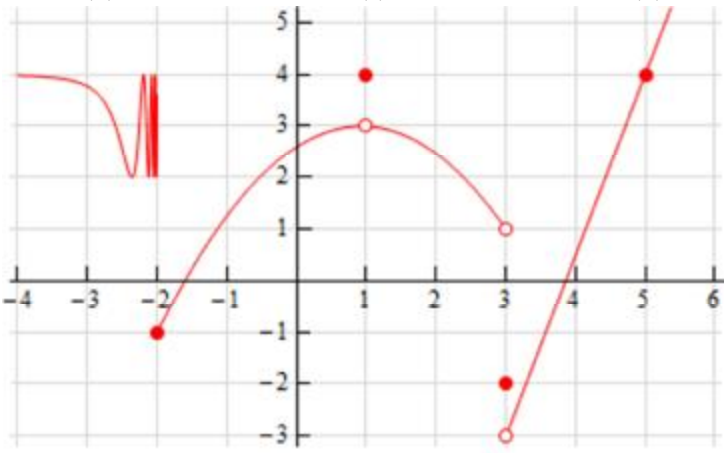
1. Below is the graph of $f(x)$. For each of the given points determine the value of $f(a)$, $\lim_{x\to a^-} f(x)$, $\lim_{x\to a^+} f(x)$, and $\lim_{x\to a} f(x)$. If any of the quantities do not exist clearly explain why.

(a) $a=-4$ **(b)** $a=-1$ **(c)** $a=2$ **(d)** $a=4$



2. Below is the graph of $f(x)$. For each of the given points determine the value of $f(a)$, $\lim_{x\to a^-} f(x)$, $\lim_{x\to a^+} f(x)$, and $\lim_{x\to a} f(x)$. If any of the quantities do not exist clearly explain why.

(a) $a=-2$ **(b)** $a=1$ **(c)** $a=3$ **(d)** $a=5$



3. Sketch a graph of a function that satisfies each of the following conditions.

$\lim_{x\to 2^-} f(x)=1$ $\lim_{x\to 2^+} f(x)=-4$ $f(2)=1$

4. Sketch a graph of a function that satisfies each of the following conditions.

$\lim_{x\to 3^-} f(x)=0$ $\lim_{x\to 3^+} f(x)=4$ $f(3)$ does not exist

$\lim_{x\to -1} f(x)=-3$ $f(-1)=2$

Limit Properties

1. Given $\lim_{x\to 8} f(x) = -9$, $\lim_{x\to 8} g(x) = 2$ and $\lim_{x\to 8} h(x) = 4$ use the limit properties given in this section to compute each of the following limits. If it is not possible to compute any of the limits clearly explain why not.

(a) $\lim_{x\to 8}\left[2f(x) - 12h(x)\right]$

(b) $\lim_{x\to 8}\left[3h(x) - 6\right]$

(c) $\lim_{x\to 8}\left[g(x)h(x) - f(x)\right]$

(d) $\lim_{x\to 8}\left[f(x) - g(x) + h(x)\right]$

2. Given $\lim_{x\to -4} f(x) = 1$, $\lim_{x\to -4} g(x) = 10$ and $\lim_{x\to -4} h(x) = -7$ use the limit properties given in this section to compute each of the following limits. If it is not possible to compute any of the limits clearly explain why not.

(a) $\lim_{x\to -4}\left[\dfrac{f(x)}{g(x)} - \dfrac{h(x)}{f(x)}\right]$

(b) $\lim_{x\to -4}\left[f(x)g(x)h(x)\right]$

(c) $\lim_{x\to -4}\left[\dfrac{1}{h(x)} + \dfrac{3 - f(x)}{g(x) + h(x)}\right]$

(d) $\lim_{x\to -4}\left[2h(x) - \dfrac{1}{h(x) + 7f(x)}\right]$

3. Given $\lim_{x\to 0} f(x) = 6$, $\lim_{x\to 0} g(x) = -4$ and $\lim_{x\to 0} h(x) = -1$ use the limit properties given in this section to compute each of the following limits. If it is not possible to compute any of the limits clearly explain why not.

(a) $\lim_{x\to 0}\left[f(x) + h(x)\right]^3$

(b) $\lim_{x\to 0}\sqrt{g(x)h(x)}$

(c) $\lim_{x\to 0}\sqrt[3]{11 + \left[g(x)\right]^2}$

(d) $\lim_{x\to 0}\sqrt{\dfrac{f(x)}{h(x) - g(x)}}$

For each of the following limits use the limit properties given in this section to compute the limit. At each step clearly indicate the property being used. If it is not possible to compute any of the limits clearly explain why not.

4. $\lim_{t\to -2}\left(14 - 6t + t^3\right)$

5. $\lim_{x\to 6}\left(3x^2 + 7x - 16\right)$

6. $\lim_{w\to 3}\dfrac{w^2-8w}{4-7w}$

7. $\lim_{x\to -5}\dfrac{x+7}{x^2+3x-10}$

8. $\lim_{z\to 0}\sqrt{z^2+6}$

9. $\lim_{x\to 10}\left(4x+\sqrt[3]{x-2}\right)$

Computing Limits

For problems 1 – 9 evaluate the limit, if it exists.

1. $\lim_{x\to 2}\left(8-3x+12x^2\right)$

2. $\lim_{t\to -3}\dfrac{6+4t}{t^2+1}$

3. $\lim_{x\to -5}\dfrac{x^2-25}{x^2+2x-15}$

4. $\lim_{z\to 8}\dfrac{2z^2-17z+8}{8-z}$

5. $\lim_{y\to 7}\dfrac{y^2-4y-21}{3y^2-17y-28}$

6. $\lim_{h\to 0}\dfrac{\left(6+h\right)^2-36}{h}$

7. $\lim_{z\to 4}\dfrac{\sqrt{z}-2}{z-4}$

8. $\lim_{x\to -3}\dfrac{\sqrt{2x+22}-4}{x+3}$

9. $\lim_{x\to 0}\dfrac{x}{3-\sqrt{x+9}}$

10. Given the function

$$f(x)=\begin{cases}7-4x & x<1\\ x^2+2 & x\ge 1\end{cases}$$

Evaluate the following limits, if they exist.

(a) $\lim_{x\to -6} f(x)$ (b) $\lim_{x\to 1} f(x)$

11. Given

$$h(z)=\begin{cases}6z & z\le -4\\ 1-9z & z>-4\end{cases}$$

Evaluate the following limits, if they exist.

(a) $\lim_{z\to 7} h(z)$ (b) $\lim_{z\to -4} h(z)$

For problems 12 & 13 evaluate the limit, if it exists.

12. $\lim_{x\to 5}\left(10+|x-5|\right)$

13. $\lim_{t\to -1}\dfrac{t+1}{|t+1|}$

14. Given that $7x\le f(x)\le 3x^2+2$ for all x determine the value of $\lim_{x\to 2} f(x)$.

15. Use the Squeeze Theorem to determine the value of $\lim_{x\to 0} x^4\sin\left(\dfrac{\pi}{x}\right)$.

Infinite Limits

For problems 1 – 6 evaluate the indicated limits, if they exist.

1. For $f(x)=\dfrac{9}{(x-3)^5}$ evaluate,

(a) $\lim_{x\to 3^-} f(x)$ **(b)** $\lim_{x\to 3^+} f(x)$ **(c)** $\lim_{x\to 3} f(x)$

2. For $h(t)=\dfrac{2t}{6+t}$ evaluate,

(a) $\lim\limits_{t\to -6^-} h(t)$ **(b)** $\lim\limits_{t\to -6^+} h(t)$ **(c)** $\lim\limits_{t\to -6} h(t)$

3. For $g(z)=\dfrac{z+3}{(z+1)^2}$ evaluate,

(a) $\lim\limits_{z\to -1^-} g(z)$ **(b)** $\lim\limits_{z\to -1^+} g(z)$ **(c)** $\lim\limits_{z\to -1} g(z)$

4. For $g(x)=\dfrac{x+7}{x^2-4}$ evaluate,

(a) $\lim\limits_{x\to 2^-} g(x)$ **(b)** $\lim\limits_{x\to 2^+} g(x)$ **(c)** $\lim\limits_{x\to 2} g(x)$

5. For $h(x)=\ln(-x)$ evaluate,

(a) $\lim\limits_{x\to 0^-} h(x)$ **(b)** $\lim\limits_{x\to 0^+} h(x)$ **(c)** $\lim\limits_{x\to 0} h(x)$

6. For $R(y)=\tan(y)$ evaluate,

(a) $\lim\limits_{y\to \frac{3\pi}{2}^-} R(y)$ **(b)** $\lim\limits_{y\to \frac{3\pi}{2}^+} R(y)$ **(c)** $\lim\limits_{y\to \frac{3\pi}{2}} R(y)$

For problems 7 & 8 find all the vertical asymptotes of the given function.

7. $f(x)=\dfrac{7x}{(10-3x)^4}$

8. $g(x)=\dfrac{-8}{(x+5)(x-9)}$

Limits At Infinity, Part I

1. For $f(x)=4x^7-18x^3+9$ evaluate each of the following limits.

(a) $\lim\limits_{x\to -\infty} f(x)$ **(b)** $\lim\limits_{x\to \infty} f(x)$

2. For $h(t)=\sqrt[3]{t}+12t-2t^2$ evaluate each of the following limits.

(a) $\lim_{t\to-\infty} h(t)$ **(b)** $\lim_{t\to\infty} h(t)$

For problems 3 – 10 answer each of the following questions.

(a) Evaluate $\lim_{x\to-\infty} f(x)$.

(b) Evaluate $\lim_{x\to\infty} f(x)$.

(c) Write down the equation(s) of any horizontal asymptotes for the function.

3. $f(x)=\dfrac{8-4x^2}{9x^2+5x}$

4. $f(x)=\dfrac{3x^7-4x^2+1}{5-10x^2}$

5. $f(x)=\dfrac{20x^4-7x^3}{2x+9x^2+5x^4}$

6. $f(x)=\dfrac{x^3-2x+11}{3-6x^5}$

7. $f(x)=\dfrac{x^6-x^4+x^2-1}{7x^6+4x^3+10}$

8. $f(x)=\dfrac{\sqrt{7+9x^2}}{1-2x}$

9. $f(x)=\dfrac{x+8}{\sqrt{2x^2+3}}$

10. $f(x)=\dfrac{8+x-4x^2}{\sqrt{6+x^2+7x^4}}$

Limits At Infinity, Part II

For problems 1 – 6 evaluate **(a)** $\lim_{x\to-\infty} f(x)$ and **(b)** $\lim_{x\to\infty} f(x)$.

1. $f\left(x\right)=\mathbf{e}^{8+2x-x^3}$

2. $f\left(x\right)=\mathbf{e}^{\frac{6x^2+x}{5+3x}}$

3. $f\left(x\right)=2\mathbf{e}^{6x}-\mathbf{e}^{-7x}-10\mathbf{e}^{4x}$

4. $f\left(x\right)=3\mathbf{e}^{-x}-8\mathbf{e}^{-5x}-\mathbf{e}^{10x}$

5. $f\left(x\right)=\dfrac{\mathbf{e}^{-3x}-2\mathbf{e}^{8x}}{9\mathbf{e}^{8x}-7\mathbf{e}^{-3x}}$

6. $f\left(x\right)=\dfrac{\mathbf{e}^{-7x}-2\mathbf{e}^{3x}-\mathbf{e}^{x}}{\mathbf{e}^{-x}+16\mathbf{e}^{10x}+2\mathbf{e}^{-4x}}$

For problems 7 – 12 evaluate the given limit.

7. $\lim\limits_{t\to-\infty}\ln\left(4-9t-t^3\right)$

8. $\lim\limits_{z\to-\infty}\ln\left(\dfrac{3z^4-8}{2+z^2}\right)$

9. $\lim\limits_{x\to\infty}\ln\left(\dfrac{11+8x}{x^3+7x}\right)$

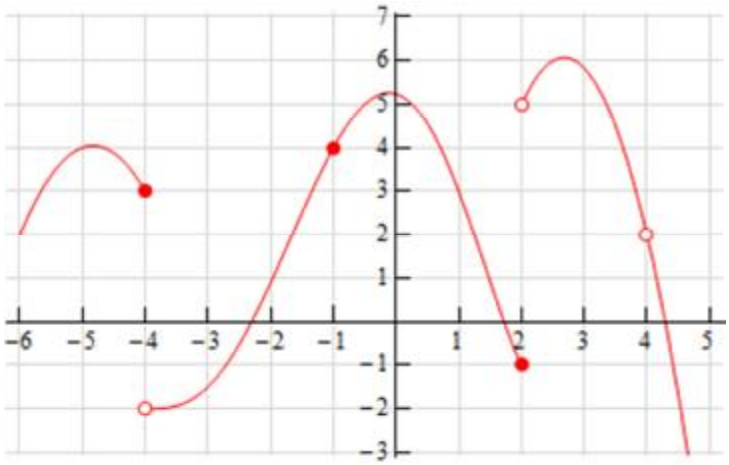
10. $\lim\limits_{x\to-\infty}\tan^{-1}\left(7-x+3x^5\right)$

11. $\lim\limits_{t\to\infty}\tan^{-1}\left(\dfrac{4+7t}{2-t}\right)$

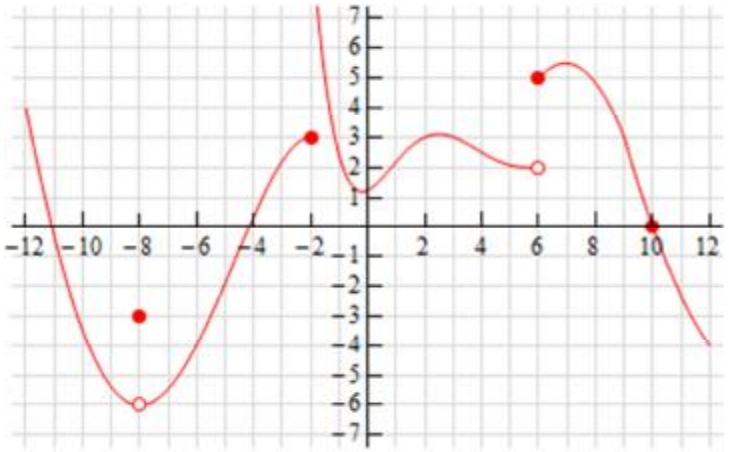
12. $\lim\limits_{w\to\infty}\tan^{-1}\left(\dfrac{3w^2-9w^4}{4w-w^3}\right)$

Continuity

1. The graph of $f(x)$ is given below. Based on this graph determine where the function is discontinuous.



2. The graph of $f(x)$ is given below. Based on this graph determine where the function is discontinuous.



For problems 3 – 7 using only Properties 1 – 9 from the Limit Properties section, one-sided limit properties (if needed) and the definition of continuity determine if the given function is continuous or discontinuous at the indicated points.

3. $f(x)=\dfrac{4x+5}{9-3x}$

(a) $x=-1$, **(b)** $x=0$, **(c)** $x=3$?

4. $g(z)=\dfrac{6}{z^2-3z-10}$

(a) $z=-2$, **(b)** $z=0$, **(c)** $z=5$?

5. $g(x)=\begin{cases} 2x & x<6 \\ x-1 & x\ge 6 \end{cases}$

(a) $x=4$, **(b)** $x=6$?

6. $h(t)=\begin{cases} t^2 & t<-2 \\ t+6 & t\ge -2 \end{cases}$

(a) $t=-2$, **(b)** $t=10$?

7. $g(x)=\begin{cases} 1-3x & x<-6 \\ 7 & x=-6 \\ x^3 & -6<x<1 \\ 1 & x=1 \\ 2-x & x>1 \end{cases}$

(a) $x=-6$, **(b)** $x=1$?

For problems 8 – 12 determine where the given function is discontinuous.

8. $f(x)=\dfrac{x^2-9}{3x^2+2x-8}$

9. $R(t)=\dfrac{8t}{t^2-9t-1}$

10. $h(z)=\dfrac{1}{2-4\cos(3z)}$

11. $y(x)=\dfrac{x}{7-\mathbf{e}^{2x+3}}$

12. $g(x)=\tan(2x)$

For problems 13 – 15 use the Intermediate Value Theorem to show that the given equation has at least one solution in the indicated interval. Note that you are NOT asked to find the solution only show that at least one must exist in the indicated interval.

13. $25-8x^2-x^3=0$ on $[-2,4]$

14. $w^2-4\ln(5w+2)=0$ on $[0,4]$

15. $4t + 10\mathbf{e}^{t} - \mathbf{e}^{2t} = 0$ on $[1,3]$

The Definition of the Limit

Problems for this section have not yet been written.