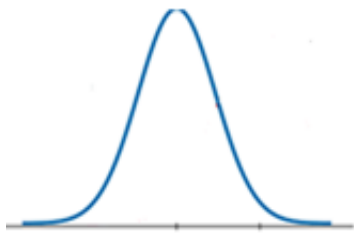


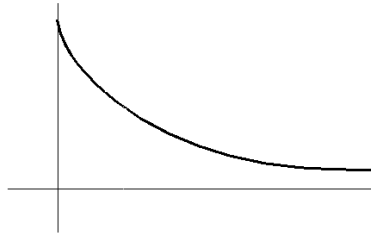
## Chapter 6: Continuous Random Variables and Probability Distributions

A continuous random variable is a random variable with an infinite number of possible values.

A continuous probability distribution is a smooth curve that serves as a model for the population distribution of a continuous variable. The total area under the curve equals 1



Normal Distribution



Exponential  
Distribution



Uniform Distribution

The characteristics of continuous random variables and continuous probability distributions:

- The outcomes are measured, not counted.
- Geometrically, the probability of an outcome is equal to an area under a mathematical curve.
- The probability that a continuous random variable assumes a specific value is zero. Instead we find the probability that the value is between two endpoints.

## The Normal Distribution

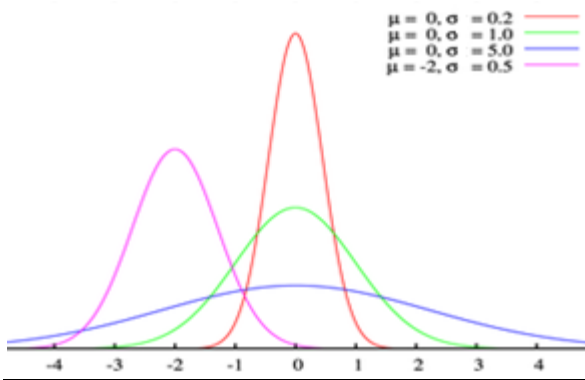
normal distribution—a bell-shaped, symmetric probability distribution; it is centered at the mean,  $\mu$ , and its shape is determined by  $\sigma$ , the standard deviation.

Human blood pressures, IQ scores, birth weights are normally distributed random variables.

The formula for the normal distribution is given by

$$f(x) = \frac{e^{-1/2(\frac{x-\mu}{\sigma})^2}}{\sigma\sqrt{2\pi}}$$

Examples of Normal Distributions



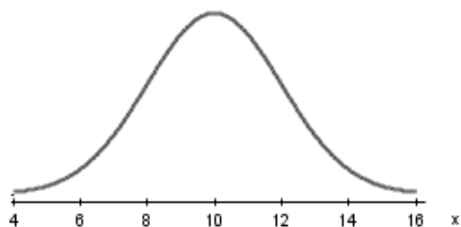
### Sketching a normal distribution

1. Draw a horizontal axis.
2. Draw a normal distribution graph.

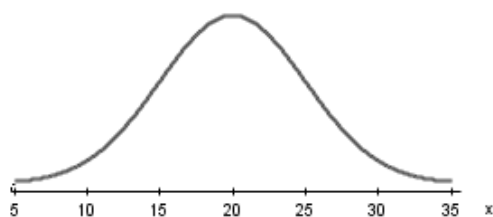


3. Label the location of the mean in the center of the graph.
4. Label the locations of the inflection points with  $\mu + \sigma$  and  $\mu - \sigma$  on the horizontal axis.
5. Using the scale set by the points  $\mu + \sigma$  and  $\mu - \sigma$ , label the locations of  $\mu + 2\sigma$  and  $\mu - 2\sigma$ .
6. Using the same scale, label the locations of  $\mu + 3\sigma$  and  $\mu - 3\sigma$ .

Example: Sketch a normal distribution with a mean of 10 and a standard deviation of 2.



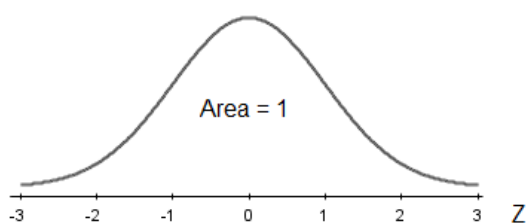
Example: Sketch a normal distribution with a mean of 20 and a standard deviation of 5.



### The Standard Normal Distribution

standard normal distribution – a normal probability distribution that has a mean of 0 and a standard deviation of 1. The total area under the curve is equal to 1.

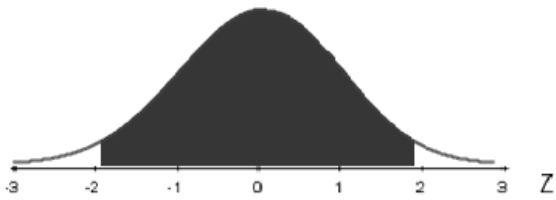
A standard normal random variable is denoted by  $z$ . We will be interested in finding the probability that  $z$  falls in some interval.



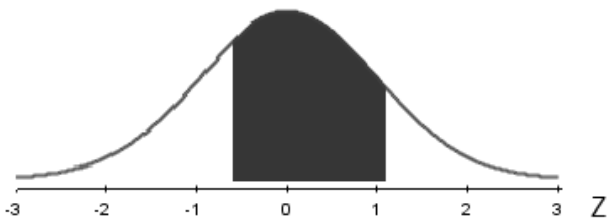
You should always draw a picture of the normal distribution and shade when finding probabilities. Then you can use your calculator to find normal probabilities.

2<sup>nd</sup> -> VARS -> 2: normalcdf -> normalcdf(left bound, right bound, mean, standard deviation)

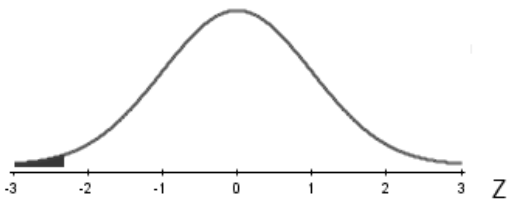
Example:  $P(-1.96 < Z < 1.96) = \text{normalcdf}(-1.96, 1.96, 0, 1) = 0.9500$



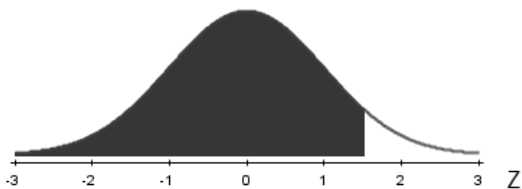
Example:  $P(-0.69 < Z < 1.09) = \text{normalcdf}(-0.69, 1.09, 0, 1) = 0.6170$



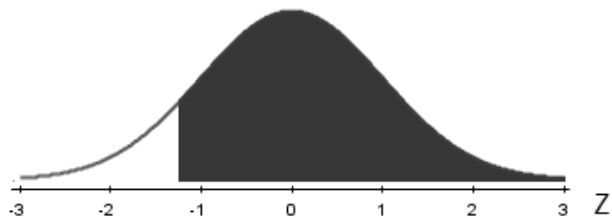
Example:  $P(Z < -2.33) = \text{normalcdf}(-9999, -2.33, 0, 1) = 0.0099$   
(Note that the left bound is negative infinity. Use a large negative number such as -9999 instead.)



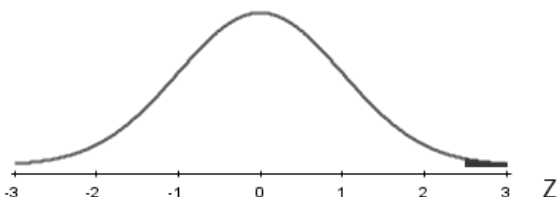
Example:  $P(Z < 1.53) = \text{normalcdf}(-9999, 1.53, 0, 1) = 0.9370$   
(Note that the left bound is negative infinity. Use a large negative number such as -9999 instead.)



Example:  $P(Z > -1.23) = \text{normalcdf}(-1.23, 9999, 0, 1) = 0.8907$   
 (Note that the left bound is positive infinity. Use a large positive number such as 9999 instead.)



Example:  $P(Z > 2.51) = \text{normalcdf}(2.51, 9999, 0, 1) = 0.0060$   
 (Note that the left bound is positive infinity. Use a large positive number such as 9999 instead.)



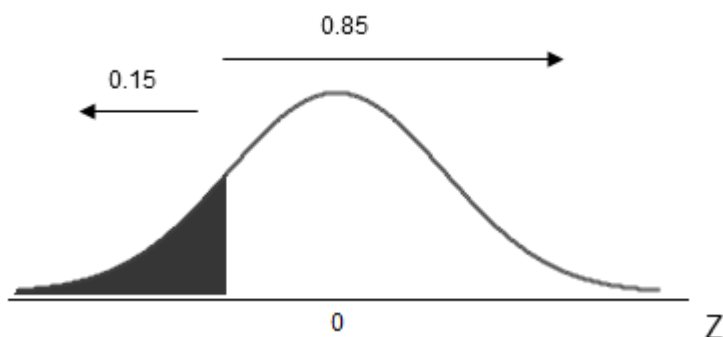
### Finding Z-scores from Probabilities

You can use your calculator to find z-scores when given probabilities.

2<sup>nd</sup> -> VARS -> 3: invNorm -> invNorm(area to the left, mean, standard deviation)

Example: What z-score separates the bottom 15% from the top 85%? In other words, what is the 15<sup>th</sup> percentile?

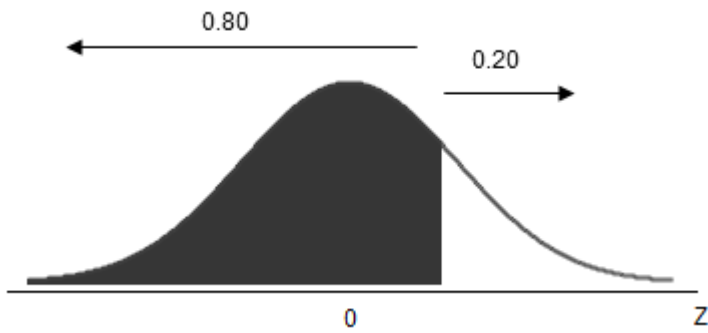
Sketch a normal curve. The z-score that we're looking for will be negative.



$z = \text{invNorm}(0.15, 0, 1) = -1.04$ . Therefore, the 15<sup>th</sup> percentile is -1.04.

Example: What z-score separates the bottom 80% from the top 20%? In other words, what is the 80<sup>th</sup> percentile?

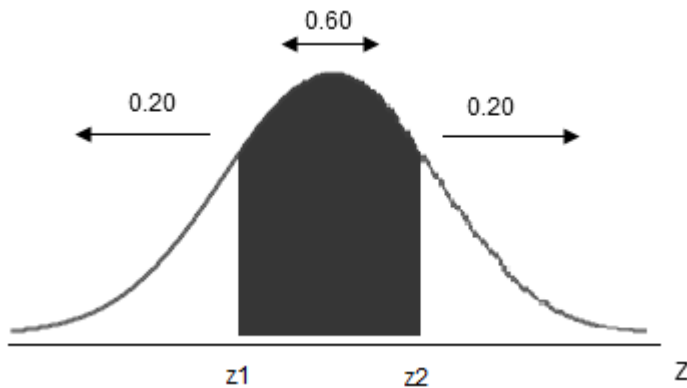
Sketch a normal curve. The z-score that we're looking for will be positive



$z = \text{invNorm}(0.80, 0, 1) = 0.84$ . Therefore, the 80<sup>th</sup> percentile is 0.84.

*Example:* The middle 60% of z-scores are between what two values?

We are looking for the two z-scores (call them  $z_1$  and  $z_2$ ) that separate the probability as shown below.



$$z_1 = \text{invNorm}(0.20, 0, 1) = -0.84$$

$$z_2 = \text{invNorm}(0.80, 0, 1) = 0.84$$

Therefore, the middle 60% of z-scores are between -0.84 and 0.84.

### Applications of Normal Distributions

Most normal distributions are not standard normal. In other words,  $X$  will be a normal random variable with a mean different than 0 and/or a standard deviation different than 1.

Any value  $x$  taken from a nonstandard normal distribution can be standardized by converting it to a z-score.

$$z = \frac{x - \mu}{\sigma}$$

If this is done to all of the values in a data set, the resulting z-scores will have a standard normal distribution.

*Example:* IQ scores are normally distributed with a mean of 100 and a standard deviation of 15. Let  $X = \text{IQ score}$ . What is the probability that a randomly selected person has an IQ score lower than 130?

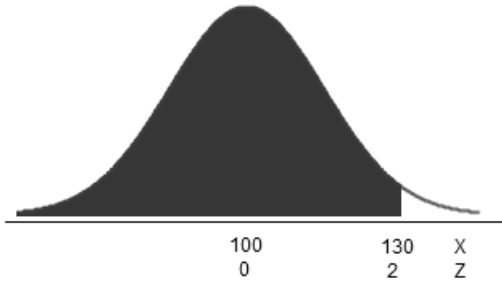
*Solution:*

We are being asked to find  $P(X < 130)$ .

We can standardize the score of 130 by using the formula and rounding to two decimal places:

$$z = \frac{x - \mu}{\sigma} = \frac{130 - 100}{15} = 2.00$$

$$P(X < 130) = P(Z < 2) = \text{normalcdf}(-9999, 2, 0, 1) = 0.9772$$



Therefore, the probability that an individual has an IQ lower than 130 is approximately 0.9772.

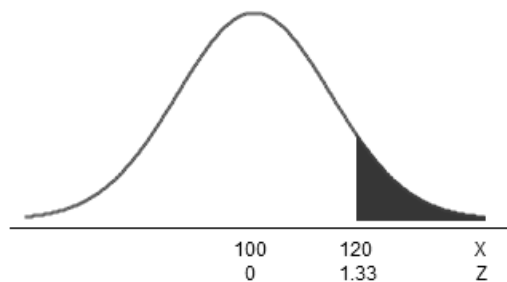
*Example:* Find the probability that an individual has an IQ higher than 120.

*Solution:* We are being asked to find  $P(X > 120)$ .

We can standardize the score of 120 by using the formula and rounding to two decimal places:

$$z = \frac{x - \mu}{\sigma} = \frac{120 - 100}{15} = 1.33$$

$$P(X > 120) = P(Z > 1.33) = \text{normalcdf}(1.33, 9999, 0, 1) = 0.0918$$



Therefore, the probability that an individual has an IQ higher than 130 is approximately 0.0918.

*Example:* Find the probability that an individual has an IQ between 90 and 120.

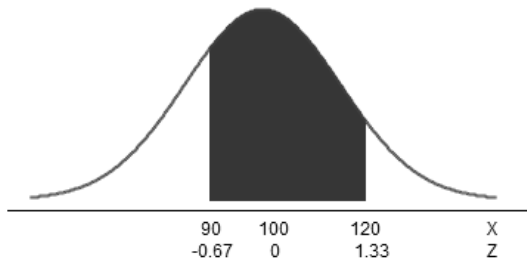
*Solution:*

We are being asked to find  $P(90 < X < 120)$ .

$$z = \frac{90 - 100}{15} = -0.67$$

$$z = \frac{120 - 100}{15} = 1.33$$

$$P(90 < X < 120) = P(-0.67 < Z < 1.33) = \text{normalcdf}(-0.67, 1.33, 0, 1) = 0.6568$$



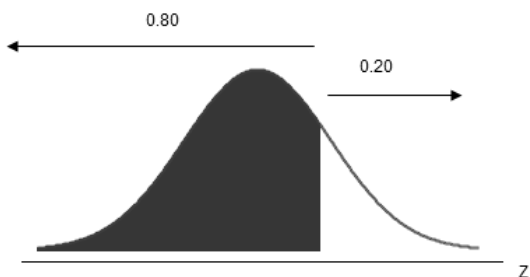
### Finding Values of X from Probabilities

Recall that you can convert a z-score to X by using the following formula:

$$X = \mu + z\sigma$$

*Example:* What IQ score separates the bottom 80% from the top 20%? In other words, find the 80<sup>th</sup> percentile for IQ scores.

*Solution:* First find the 80<sup>th</sup> percentile in terms of a z-score, and then convert that to an IQ score (X).



$z = \text{invNorm}(0.80, 0, 1) = 0.84$ . Therefore, the 80<sup>th</sup> percentile is  $z = 0.84$ .

Now convert to an IQ.

$$X = \mu + z\sigma = 100 + 0.84 \cdot 15 = 112.6$$

The 80<sup>th</sup> percentile for IQ scores is 112.6.

*Example:* The middle 60% of IQs fall between what two values?

*Solution:* Previously in the notes, we found that the middle 60% of z-scores fall between -0.84 and 0.84. Convert these z-scores to IQ scores.

$$X = \mu + z\sigma = 100 - 0.84*15 = 87.4$$

$$X = \mu + z\sigma = 100 + 0.84*15 = 112.6$$

Therefore, the middle 60% of IQ scores fall between 87.4 and 112.6.