Practice Test 4 Solutions

Precalculus

1. domain: $(-\infty, \infty)$

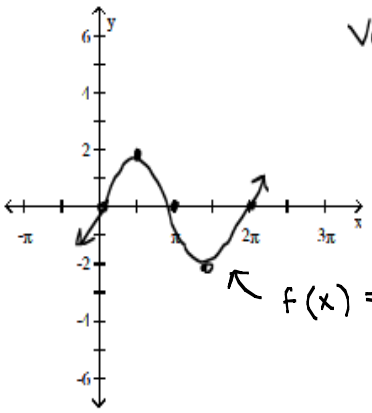
   range: $[-1, 1]$

2. $\sec x = \frac{1}{\cos x}$. Secant will be undefined at values of x that make the the denominator equal to 0.

   domain: all real numbers except $\ldots -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$ (or $\frac{\pi}{2} + n\pi$, where n is an integer)

   range: $(-\infty, -1] \cup [1, \infty)$

3. $\cot x = \frac{\cos x}{\sin x}$. It will be undefined at values of x that make the denominator equal to 0, such as $x = 0, \pi, 2\pi, \ldots$ (or $x = n\pi$, where n is an integer)

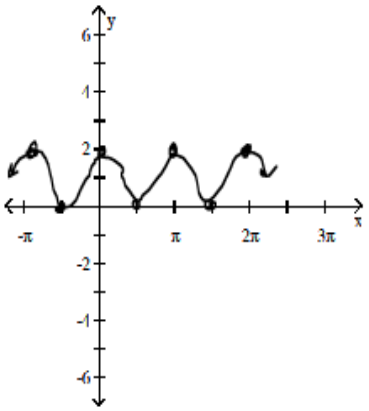
4. 

   Vertical stretch by a factor of 2.

   amplitude = 2

   period = 2π

   $f(x) = 2\sin x$

5. 

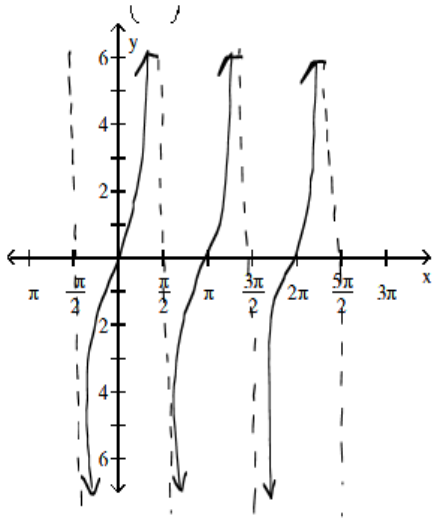
   Horizontal compression by a factor of 1/2.

   Shift up one unit.

   amplitude = 1

   period = π

6.

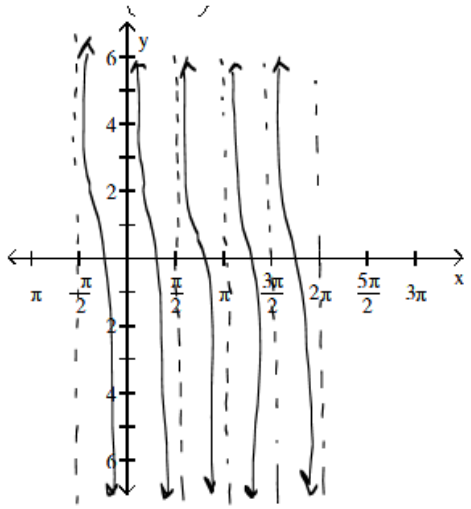


period: π

x-intercepts: $\ldots -\pi, 0, \pi, \ldots$

asymptotes: $\ldots -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$

7.



Horizontal compression by a factor of $\frac{1}{2}$.

period: $\frac{\pi}{2}$

x-intercepts: $\ldots -\frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4} \ldots$

asymptotes: $\ldots -\frac{\pi}{2}, 0, \frac{\pi}{2}, \ldots$

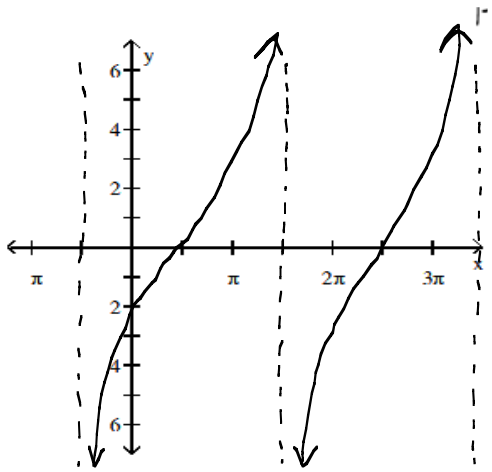
To find the asymptotes of $f(x) = \cot(2x)$, use the fact that two of the asymptotes of cotangent are $x = 0$ and $x = \pi$.

$0 < 2x < \pi$

$0 < x < \frac{\pi}{2}$

Therefore, two of the asymptotes of $\cot(2x)$ are $x = 0$ and $x = \frac{\pi}{2}$

8.



period: 2π

x-intercepts: $\ldots \frac{\pi}{2}, \frac{5\pi}{2}, \ldots$

asymptotes: $\ldots -\frac{\pi}{2}, \frac{3\pi}{2}, \ldots$

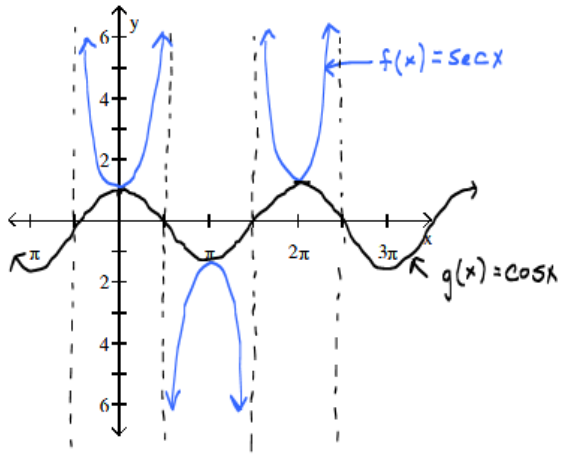
To find the asymptotes of $f(x) = \tan\left(\frac{1}{2}x - \frac{\pi}{4}\right)$, use the fact that two of the asymptotes of tangent are $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$.

$$-\frac{\pi}{2} < \frac{1}{2}x - \frac{\pi}{4} < \frac{\pi}{2}$$

$$-\frac{\pi}{4} < \frac{1}{2}x < \frac{3\pi}{4}$$

$$-\frac{\pi}{2} < x < \frac{3\pi}{2}$$

9.

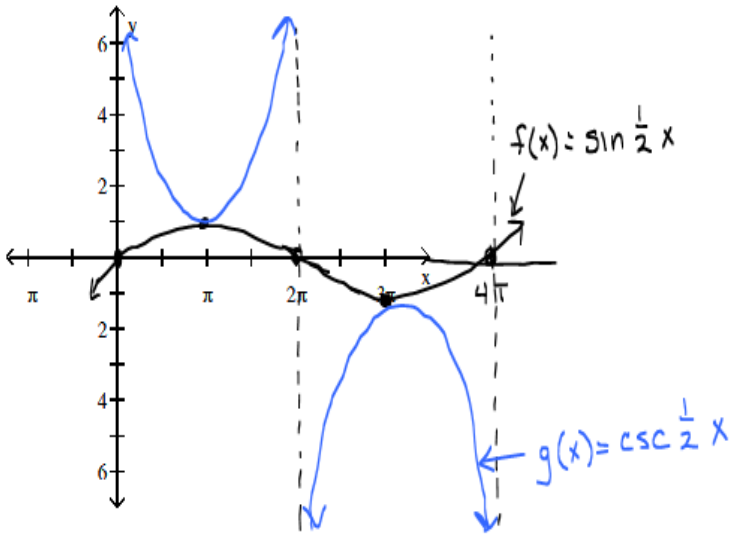


period: 2π

no x-intercepts

asymptotes: $\ldots -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2} \ldots$

10.



$\csc\frac{1}{2}x = \frac{1}{\sin\frac{1}{2}x}$

Graph $f(x) = \sin\frac{1}{2}x$ first

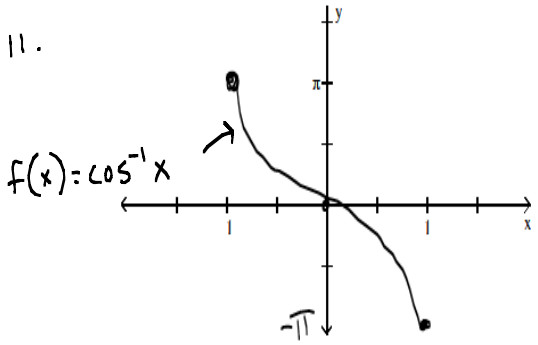
$f(x) = \sin\frac{1}{2}x$ is stretched horizontally by a factor of 2.

period: 4π

no x-intercepts

asymptotes: $\ldots 0, 2\pi, 4\pi$

11.



domain: $[-1, 1]$

range: $[-\pi, \pi]$

12. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$

13. $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$

14. $\tan^{-1}(-1) = -\frac{\pi}{4}$

15. $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$

16. $\sin^{-1}(0.6) \approx 0.64$

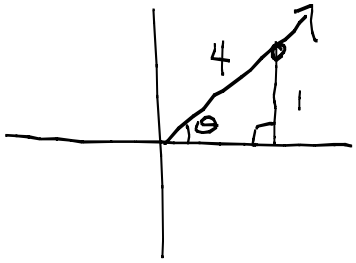
17. $\cos^{-1}\left(-\frac{\sqrt{5}}{5}\right) \approx 2.03$

18. $\sin^{-1}\left(\sin\left(\frac{\pi}{4}\right)\right) = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

19. $\cos^{-1}\left(\cos\left(\frac{5\pi}{4}\right)\right) = \cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$

20. Let $\theta = \sin^{-1}\frac{1}{4} \longrightarrow \sin\theta = \frac{1}{4}$

θ is a first quadrant angle.



$\sin^2\theta + \cos^2\theta = 1$ (Pythagorean Identity)

$\left(\frac{1}{4}\right)^2 + \cos^2\theta = 1$

$\frac{1}{16} + \cos^2\theta = \frac{16}{16}$

$\cos^2\theta = \frac{15}{16}$

$\cos\theta = \frac{\sqrt{15}}{4}$ ← Take the positive square root because θ is a first quadrant angle.