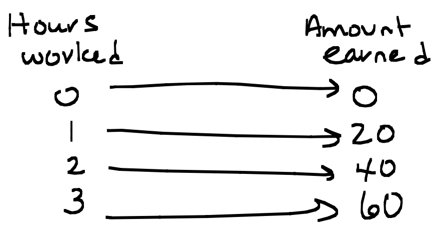
Section 1.1
Functions

Objectives

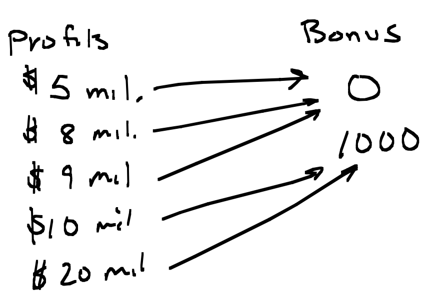
- Define a function.
- Evaluate a function at a certain value.
- Interpret tabular and graphical representations of a function.
- Define the domain and range of a function.

Relation—a correspondence between two sets.

Example: Suppose someone makes $20 per hour.

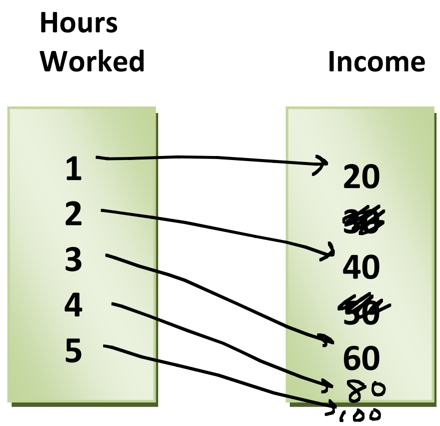


Example: Suppose I work for a company whose bonus plan is this: If the yearly profits are less than $10 million, there is no bonus. If the yearly profits are $10 million or more, all employees receive a $1000 bonus.

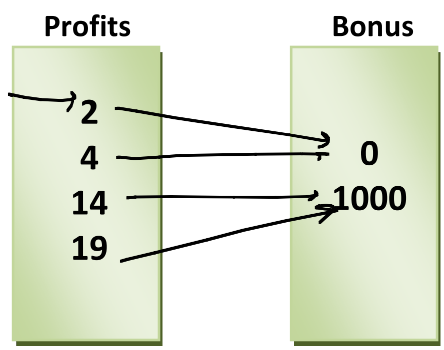


Function—a relation that establishes a correspondence between a set of input values and a set of output values in such a way that for each allowable input value, there is exactly one corresponding output ~~variable.~~ value

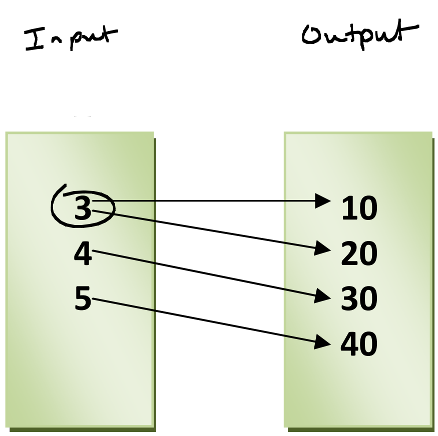
You can use a mapping to determine whether a relation is a function.



Is a function



Is a function



Not a function.

Tabular Representation of a Function

Example: Does the following table represent a function?

Input	Output
3	9
4	10
5	11
6	12

yes

Example: Does the following table represent a function?

3	9
3	10
5	11
6	12

no

(3, 9) (3, 10) (5, 11) (6, 12)

Find the Value of a Function

Functions are denoted by letters such as f, F, g, G and others.

Function notation: $f(x)$ (Not f times x)

This is read as "*f* of *x*" and is the value of the function *f* at the number *x*.

$f(x) = x + 20$ Take an input value, add 20

$f(x) = 5x$ Take an input value, multiply by 5

Example: For the function f defined by $f(x) = 20x$, evaluate

$f(x) = 20x$

a. $f(2) = 20 \cdot 2 = 40$

$f(x) = 20x$

b. $f(10) = 20 \cdot 10 = 200$

$f(x) = 20x$

c. $f(2x) = (20)(2x) = 40x$

$f(5x) = (20)(5x) = 100x$

$f(x^2) = (20)(x^2) = 20x^2$

d. $f(x+3) = 20(x+3) = 20x + 60$

Example: For the function f defined by $f(x) = 2x^2 + 3$, evaluate

$f(x) = 2x^2 + 3$

a. $f(5)$ $= 2(5)^2 + 3 = 50 + 3 = 53$

b. $f(-3)$ $= 2(-3)^2 + 3 = 2 \cdot 9 + 3 = 21$

$f(x) = 2x^2 + 3$

c. $f(-x)$ $= 2(-x)^2 + 3 = 2x^2 + 3$

$(x+2)(x+2)$

FOIL

d. $f(x + 2)$ $= 2(x+2)^2 + 3 = 2(x^2 + 4x + 4) + 3$

$= 2x^2 + 8x + 8 + 3$

$= 2x^2 + 8x + 11$

Finding the Domain of a Function

Domain of f: The set of all input values for which the function will produce a real number.

Range of f: The set of all output values that are possible for the given domain of the function.

Use interval notation $[0, \infty)$

Example: Find the domain of $f(x) = x + 1$

What am I allowed to plug in for x?

x	$f(x)$
0	1
1	2
2	3
⋮	

Domain: All real numbers

$(-\infty, \infty)$

Range: $(-\infty, \infty)$

Example: Find the domain of $f(x) = \sqrt{x}$.

Domain: $[0, \infty)$

x	$f(x)$
0	0
1	1
~~-2~~	

Range: $[0, \infty)$ All positive reals

Example: Find the domain of $f(x) = \dfrac{1}{x}$

Domain: Every number except 0.

$(-\infty, 0) \cup (0, \infty)$

Range: $(-\infty, 0) \cup (0, \infty)$

x	$f(x)$
2	$1/2$
3	$1/3$
$\frac{1}{2}$	2
$\frac{1}{3}$	3
$-\frac{1}{2}$	-2
$-\frac{1}{3}$	-3
	~~0~~

Example: Find the domain of $f(x) = \dfrac{x}{x-1}$

$x - 1 \neq 0$

$x \neq 1$

1 is excluded from the domain

$(-\infty, 1) \cup (1, \infty)$

x	$f(x)$
0	$\frac{0}{0-1} = 0$
-1	$\frac{-1}{-1-1} = \frac{-1}{-2} = \frac{1}{2}$
1	$\cancel{\frac{1}{1-1} = \frac{1}{0}}$

Example: Find the domain of $f(x) = \sqrt{2x - 1}$

$2x - 1 \geq 0$
$+1 \quad\ +1$

$\dfrac{\cancel{2}x}{\cancel{2}} \geq \dfrac{1}{2}$

$x \geq \frac{1}{2}$

domain: $[1/2, \infty)$

x	$f(x)$
$1/2$	0
1	1

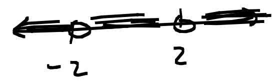
Example: Find the domain of $f(x) = \frac{x^2}{x^2-4}$

$x^2 - 4 \neq 0$

$(x+2)(x-2) \neq 0$

$x+2 \neq 0 \qquad x-2 \neq 0$

$x \neq -2 \qquad x \neq 2$



domain: $(-\infty,-2) \cup (-2,2) \cup (2,\infty)$

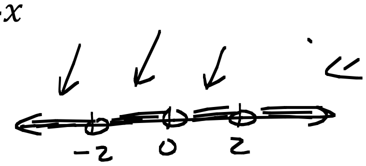
Example: Find the domain of $\frac{x+4}{x^3-4x}$

$x^3 - 4x \neq 0$

$x(x^2-4) \neq 0$

$x(x+2)(x-2) \neq 0$

$x \neq 0 \qquad x \neq -2 \qquad x \neq 2$



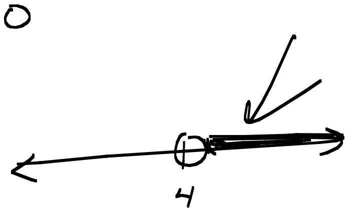
domain: $(-\infty,-2) \cup (-2,0) \cup (0,2) \cup (2,\infty)$

Example: Find the domain of $\frac{x}{\sqrt{x-4}}$

x	$f(x)$
$\cancel{4}$	$\cancel{\frac{4}{\sqrt{4-4}} = \frac{4}{\sqrt{0}}} = \frac{4}{0}$

$x - 4 > 0$

$x > 4$



domain: $(4,\infty)$

Homework: Section 1.1: 3-13 (odd), 23-33 (odd), 41, 43-55 (odd)