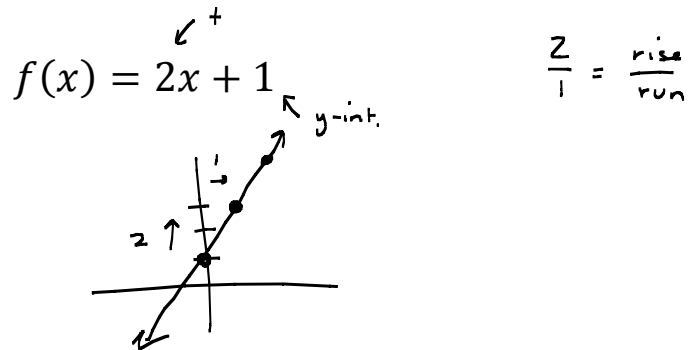


Section 1.5: Intersection of Lines and Linear Inequalities

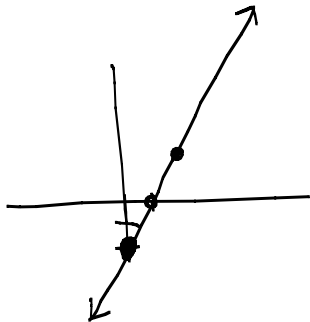
Objectives

- Find the point of intersection of two lines both algebraically and by graphing.
- Solve linear inequalities both algebraically and by graphing.

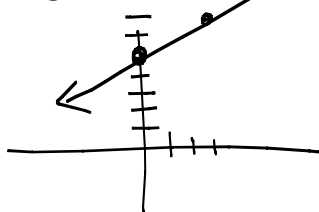
Example: Sketch the graph of the line.



$$f(x) = 2x - 2$$

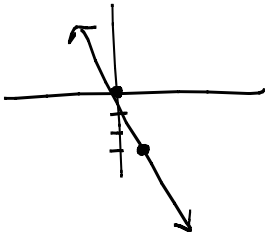


$$f(x) = \frac{2}{3}x + 5$$



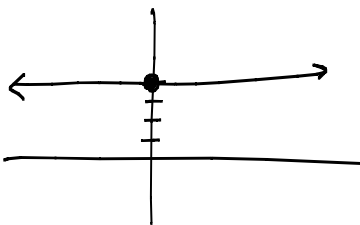
$$f(x) = -3x$$

$$\frac{\text{rise}}{\text{run}} = \frac{-3}{1}$$



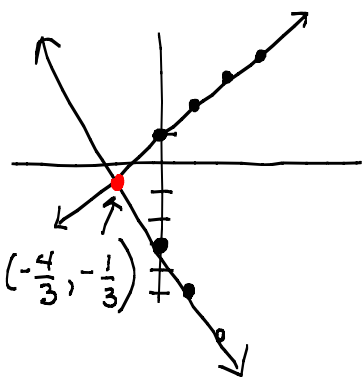
$$f(x) = 4 \quad \leftarrow \begin{array}{l} \text{y-int.} \\ \text{slope} = 0 \end{array}$$

x	f(x)
0	4
1	4
2	4
3	4



Example: Find the point of intersection for each pair of lines both algebraically and graphically.

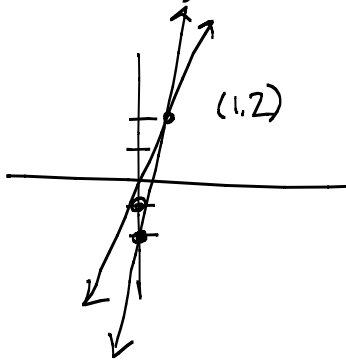
$$y = x + 1; y = -2x - 3$$



$$\begin{array}{l} \swarrow \\ y = x + 1 \\ \text{slope} = 1 \quad \frac{1}{1} \\ \text{y-int} = 1 \end{array}$$

$$\begin{array}{l} y = -2x - 3 \\ \text{slope} = -2 \quad \frac{-2}{1} \quad \frac{\text{rise}}{\text{run}} \\ \text{y-int} = -3 \end{array}$$

$$y = 3x - 1; y = 4x - 2$$



Algebraically

$$\begin{array}{r} 3x - 1 = 4x - 2 \\ -3x \quad -3x \end{array}$$

$$\begin{array}{r} -1 = x - 2 \\ +2 \quad +2 \\ \hline 1 = x \end{array}$$

$$1 = x$$

Plug $x=1$ into either equation
 $y = 2$

$$y = \frac{3}{4}x + 2; y = 3x + 1$$

$$\frac{3}{4}x + 2 = 3x + 1$$

↳ LCD = 4 mult. every term by 4

$$\begin{array}{r} 3x + 8 = 12x + 4 \\ -3x \quad -3x \end{array}$$

$$\begin{array}{r} 8 = 9x + 4 \\ -4 \quad -4 \end{array}$$

$$\frac{4}{9} = \frac{9x}{9} \rightarrow x = \frac{4}{9}$$

$$y = 3\left(\frac{4}{9}\right) + 1 = \frac{4}{3} + 1 = \frac{7}{3}$$

$$y = \frac{1}{3}x + 5; y = -\frac{3}{4}x - 2$$

$$y = 0.25x + 4; y = -0.3x + 2$$

$$\underbrace{x + y = 1}; \underbrace{2x + 3y = 7} \quad \left. \begin{array}{l} \downarrow \\ \downarrow \end{array} \right\} \text{Solve for } y \text{ first}$$

$$-3x + y = -4; 2x - y = 1$$

Review: Solving Inequalities

$$7x - 12 \leq 2$$
$$+ 12 \quad + 12$$

$$\frac{7x}{7} \leq \frac{14}{7}$$

$$x \leq 2$$

$$-5x + 12 \leq 17$$
$$- 12 \quad - 12$$

$$\frac{-5x}{-5} \leq \frac{5}{-5}$$

$$x \geq -1$$

When you mult. or divide
an inequality by a negative
number, reverse the direction
of the inequality.

$$-4 \leq 3x - 2 < 7$$
$$+ 2 \quad + 2 \quad + 2$$

← combined
inequality - an inequality
with 3 parts.

$$\frac{-2}{3} \leq \frac{3x}{3} < \frac{9}{3}$$

$$\frac{-2}{3} \leq x < 3$$

$$\left[-\frac{2}{3}, 3\right)$$

$$\frac{1}{2} \leq \frac{-2x - 1}{3} < 6$$

Example: Solve the inequalities algebraically and graphically.

$$\begin{array}{r} \downarrow \\ 4x + 1 \leq 3x + 2 \\ -3x \quad -3x \end{array}$$

$$\begin{array}{r} x + 1 \leq 2 \\ -1 \quad -1 \end{array}$$

$$x \leq 1 \quad (-\infty, 1]$$

↑
0

$$\begin{array}{r} \downarrow \quad \downarrow \\ 2 - x \geq 3x + 5 \\ +x \quad +x \end{array}$$

$$\begin{array}{r} 2 \geq 4x + 5 \\ -5 \quad -5 \end{array}$$

$$-3 \geq 4x$$

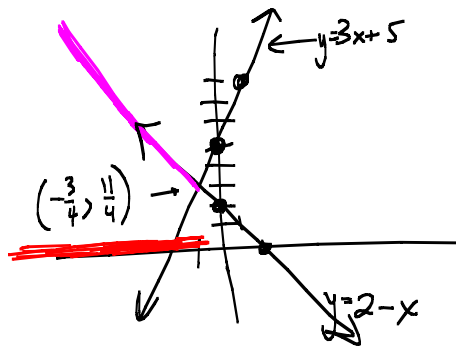
$$-\frac{3}{4} \geq x$$

$$x \leq -\frac{3}{4} \quad (-\infty, -\frac{3}{4}]$$

$$y = 4x + 1 \leftarrow$$

$$y = 3x + 2 \leftarrow$$

Where is the value of the first function less than the value of the second



$$4(x + 2) \geq x - 5$$

Cost and Revenue

Example: To manufacture boxes, it costs \$750 (the fixed cost) plus \$2 for each box produced. The boxes are then sold for \$4 each.

Find the break-even point.

$$f(x) = 750 + 2x \quad \text{Cost function}$$

$$f(x) = 4x \quad \text{Revenue function}$$

$$750 + 2x = 4x$$

$$\begin{array}{r} -2x \\ -2x \end{array}$$

$$750 = 2x$$

$$375 = x \quad \leftarrow \text{break-even point}$$

Example: To operate a coffee booth at the mall, it costs \$500 (the fixed cost) plus \$6 for each pound of coffee bought at wholesale price. The coffee is then sold at \$10 per pound.

Find the break-even point.

Example: A 2009 Toyota Prius costs \$22,000 and gets 46 miles per gallon. A 2009 Toyota Corolla costs \$16,150 and gets 35 miles per gallon. Assume that gasoline costs \$4 per gallon.

How many miles would you have to drive for the total cost of the Prius to be the same as that of the Corolla?

Cost of Prius : $f(x) = 22,000 +$

Cost of Corolla :

$x = \text{miles driven}$

x	$f(x)$
1	

$$\left(\frac{1 \text{ mile}}{46 \text{ mile}}\right) \frac{\$4}{46 \text{ mile}} = \frac{2}{23}$$

Example: Thrifty Car Rentals charges a flat rate of \$38 per day to rent a car, with unlimited mileage. Budget Rentals charges \$26 per day plus \$0.15 per mile.

How many miles must be driven so that it is more economical to rent a car at Thrifty Rentals?

Homework:

Section 1.5:

- 13 – 17 (odd—use calculator when solving graphically)
- 19 --37 (odd)
- 43 – 47 (odd—use calculator when solving graphically)
- 49 – 67 (odd)
- 74, 75, 77, 78, 83, 84