Section 2.1: Coordinate Geometry: Distance, Midpoints, and Circles

Objectives

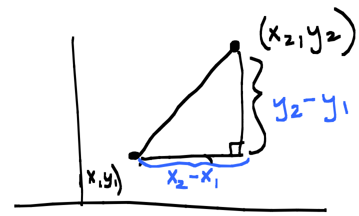
- Use the distance formula
- Use the midpoint formula
- Use the formula for the equation of a circle

The Distance Formula

Point 1 Point 2

The distance between two points (x_1, y_1) and (x_2, y_2).

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

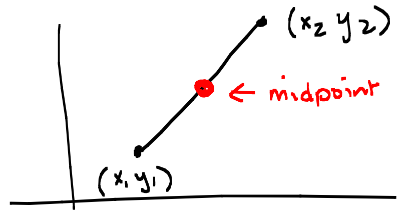


The Midpoint Formula

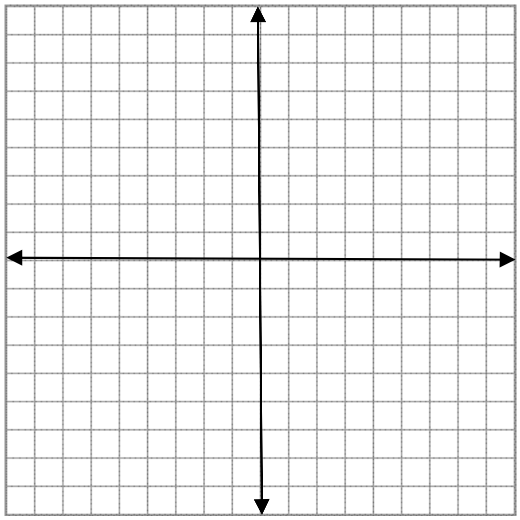
The midpoint of the line segment joining the points (x_1, y_1) and (x_2, y_2) has the coordinates

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

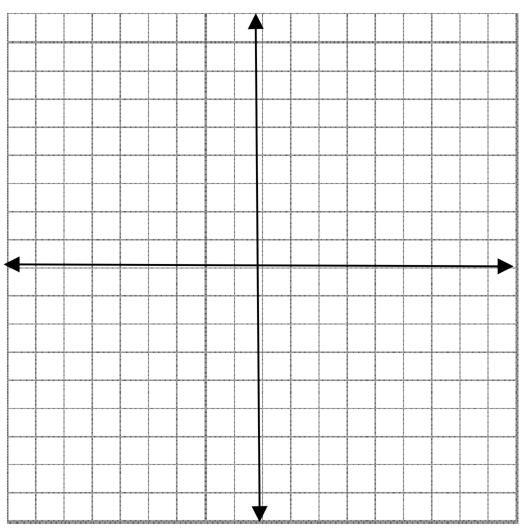
average the x coordinates average the y coordinates



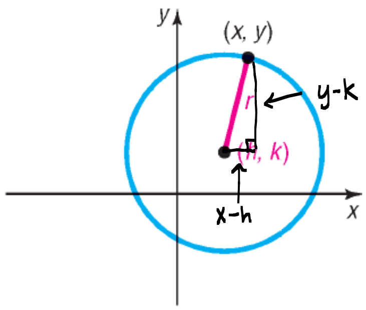
Example: Find the distance between the points (-4, 5) to (3, 2). Also, find the midpoint.



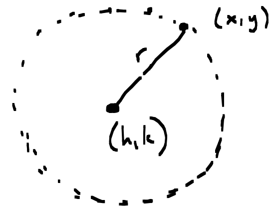
Example: Find the distance between the points (-2, 7) to (3, -2). Also, find the midpoint.



Circles



The set of all points (x,y) that are a fixed distance r from the center (h,k)



<u>The equation of a circle in standard form</u> with center at (h,k) and radius r is

$$(\mathrm{x}-\mathrm{h})^2+(\mathrm{y}-\mathrm{k})^2 = \mathrm{r}^2$$

Comes directly from distance formula.

(0,0)

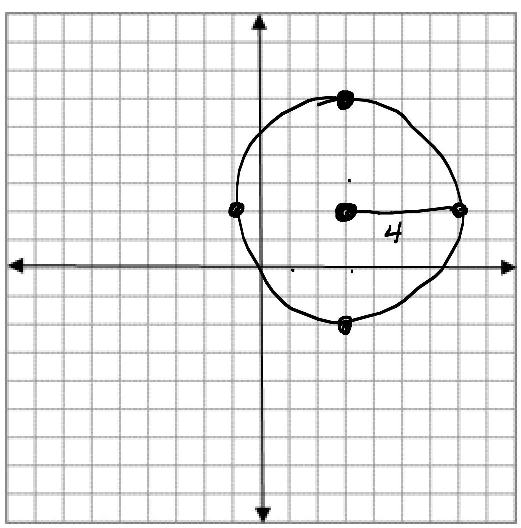
If the center is at the origin, the equation becomes $\underline{\mathrm{x}^2+\mathrm{y}^2 = \mathrm{r}^2}$

Example: Write the standard form of the equation of a circle with center (3, 2) and radius 4. Sketch the circle.

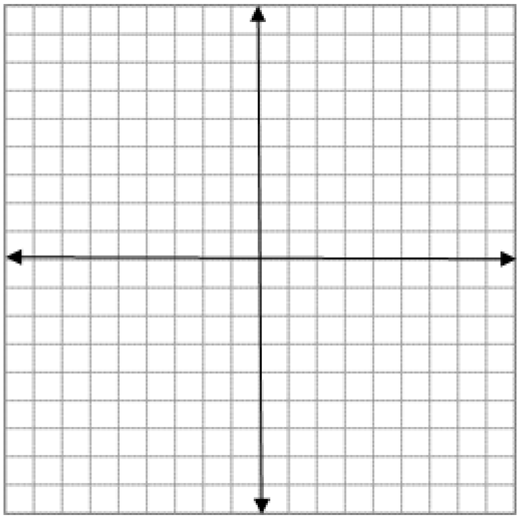
h k r

$$(x-h)^2+(y-k)^2=r^2$$

$$(x-3)^2+(y-2)^2=16$$



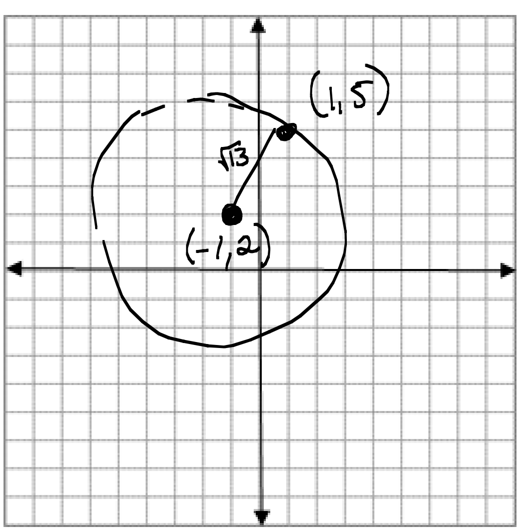
Example: Write the standard form of the equation of a circle with center (1, -2) and radius 3. Sketch the circle.



Finding the Equation of a Circle given a Point on the Circle

$h \quad k$

Example: Write the standard form of the equation of the circle with center at $(-1, 2)$ and containing the point $(1, 5)$. Sketch the circle.



$$(x-h)^2 + (y-k)^2 = r^2$$

Use distance formula to get r.

$$r = \sqrt{(-1-1)^2 + (2-5)^2}$$

$$= \sqrt{(-2)^2 + (-3)^2}$$

$$= \sqrt{4 + 9}$$

$$= \sqrt{13}$$

$$(x+1)^2 + (y-2)^2 = 13$$

The General Form of the Equation of a Circle

If we eliminate the parentheses from the standard form of the equation of a circle, we can write it in general form:

$x^2 + y^2 + 4x + 2y + 6 = 0$

$x^2 + y^2 + ax + by + c = 0.$

Example: Write $(x-1)^2 + (y+3)^2 = 4$ in general form.

$$(x-1)(x-1) + (y+3)(y+3) = 4$$

$$x^2 \underbrace{- x - x}_{-2x} + 1 + y^2 + \underbrace{3y + 3y}_{6y} + 9 = 4$$

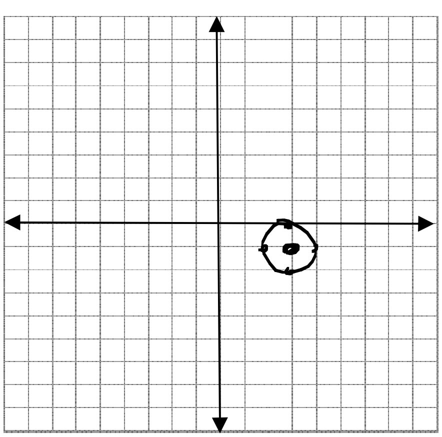
$$x^2 + y^2 - 2x + 6y + 6 = 0$$

Graphing a Circle Whose Equation is in General Form

Use the method of completing the square to put the equation in standard form so that we can identify its center and radius.

Put x terms together, y terms together

Example: Graph the equation $x^2 + y^2 - 6x + 2y + 9 = 0.$



$$\underbrace{x^2 - 6x + 9}_{(x-3)(x-3)} + \underbrace{y^2 + 2y + 1}_{(y+1)(y+1)} = -9 + 9 + 1$$

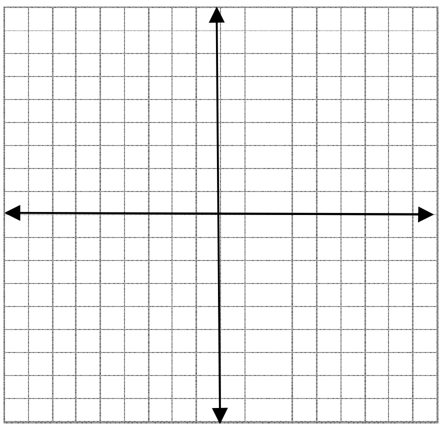
$$(x-3)^2 + (y+1)^2 = 1$$

r^2

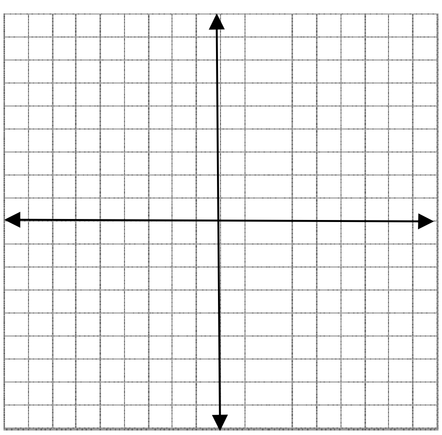
Center: $(3, -1)$

Radius $= 1$

Example: Graph the equation $x^2 + y^2 - 10x + 4y + 20 = 0$.



Example: Graph the equation $x^2 + y^2 + 8x - 2y + 8 = 0$.



Graphing a Circle on a Graphing Calculator

Solve the equation for y, using the square root method.

Make sure that you have a square screen by pressing Zsquare.

Graph the top half and the bottom half.

Example: Graph the equation $x^2 + y^2 = 4$ on a graphing calculator.

Example: Graph the equation $(x-1)^2 + (y+3)^2 = 4$ on a graphing calculator.

Homework

Section 2.1: 7 – 13 (odd), 19 – 33 (odd), 43 – 53 (odd)