

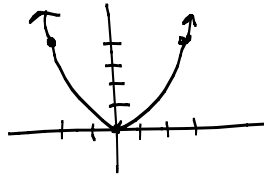
Section 2.3 Transformations of the Graph of a Function

Objectives

1. Graph Functions Using Vertical and Horizontal Shifts.
2. Graph Functions Using Vertical and Horizontal Compressions and Stretches.
3. Graph Functions Using Reflections about the x-Axis and y-Axis.

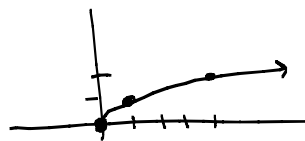
$$f(x) = x^2$$

x	f(x)
-2	4
-1	1
0	0
1	1
2	4



$$f(x) = \sqrt{x}$$

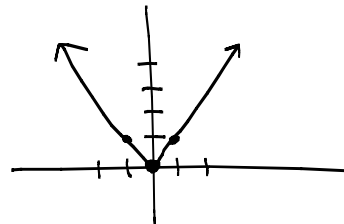
x	f(x)
0	0
1	1
4	2



"hook"

$$f(x) = |x|$$

x	f(x)
-2	2
-1	1
0	0
1	1
2	2



"V"

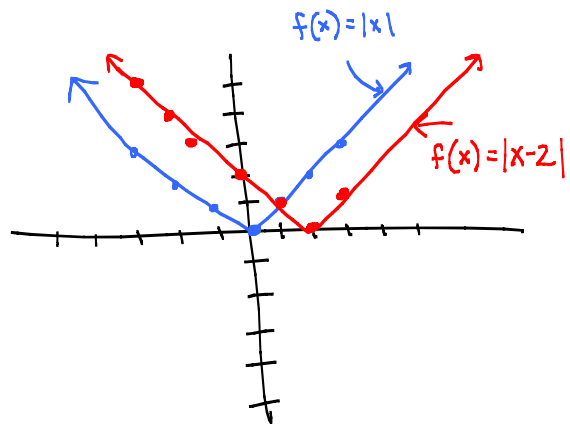
Vertical Shifts of the Graph of $f(x)$

Let f be a function and c be a positive constant.

- The graph of $g(x) = f(x) + c$ is the graph of $f(x)$ shifted c units upward.
- The graph of $g(x) = f(x) - c$ is the graph of $f(x)$ shifted c units downward.

Example: Compare $f(x) = |x|$ and $f(x) = |x - 2|$

x	$f(x) = x $	$f(x) = x - 2 $
-3	3	5
-2	2	4
-1	1	3
0	0	2
1	1	1
2	2	0
3	3	1



Example: Compare $f(x) = |x|$ and $f(x) = |x + 1|$

x	$f(x) = x $	$f(x) = x + 1 $

Horizontal Shifts of the Graph of $f(x)$

Let f be a function and c be a positive constant.

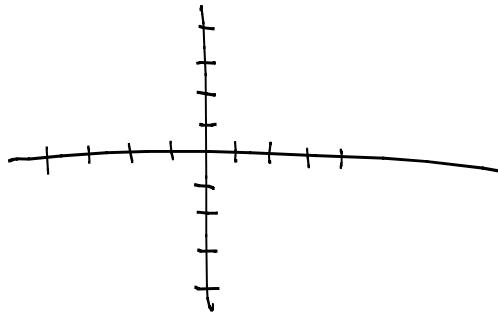
- The graph of $g(x) = f(x - c)$ is the graph of $f(x)$ shifted c units to the right.
- The graph of $g(x) = f(x) - c$ is the graph of $f(x)$ shifted c units to the left.

Combining Vertical and Horizontal Shifts



Example: Graph the function $f(x) = |x + 3| - 2$ using transformations.

x	$f(x) = x $	$f(x) = x + 3 $	$f(x) = x + 3 - 2$



Vertical Scalings and Reflections Across the Horizontal Axis

Example: Compare the graphs of $f(x) = x^2$, $f(x) = 2x^2$, and $f(x) = \frac{1}{2}x^2$

x	$f(x) = x^2$	$f(x) = 2x^2$	$f(x) = \frac{1}{2}x^2$

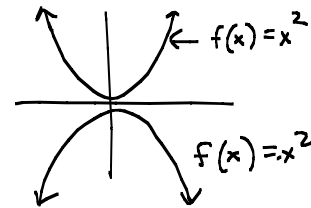
Vertical Scalings of the Graph of $f(x)$

Let f be a function and c be a positive constant.

- If $c > 1$, the graph of $g(x) = cf(x)$ is the graph of $f(x)$ stretched vertically away from the x -axis, with the y -coordinates of $g(x)$ multiplied by c .
- If $c < 1$, the graph of $g(x) = cf(x)$ is the graph of $f(x)$ compressed vertically toward the x -axis, with the y -coordinates of $g(x)$ multiplied by c .

Example: Compare the graph of $f(x) = x^2$ to $f(x) = -x^2$.

x	$f(x) = x^2$	$f(x) = -x^2$
-2	4	-4
-1	1	-1
0	0	0
1	1	-1
2	4	-4



Reflections of the Graph of $f(x)$ Across the x -Axis

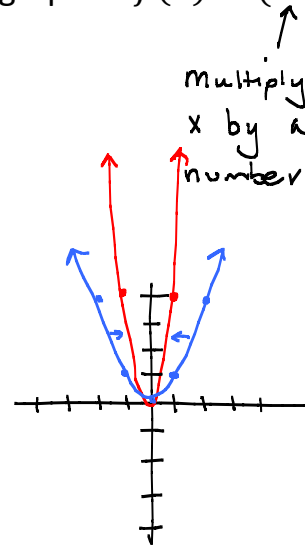
Let f be a function.

- The graph of $g(x) = -f(x)$ is the graph of $f(x)$ reflected across the x -axis.

Horizontal Scalings and Reflections Across the Vertical Axis

Example: Compare the graph of $f(x) = x^2$ to the graph of $f(x) = (2x)^2$

x	$f(x) = x^2$	$f(x) = (2x)^2$
-2	4	16
-1	1	4
0	0	0
1	1	4
2	4	16



Example: Compare the graph of $f(x) = x^2$ to the graph of $f(x) = \left(\frac{1}{2}x\right)^2$

x	$f(x) = x^2$	$f(x) = \left(\frac{1}{2}x\right)^2$

Horizontal Scaling of the Graph of $f(x)$

Let f be a function and c be a positive constant.

- If $c > 1$, the graph of $g(x) = f(cx)$ is the graph of $f(x)$ compressed horizontally toward the y -axis, scaled by a factor of $\frac{1}{c}$.
- If $0 < c < 1$, the graph of $g(x) = f(cx)$ is the graph of $f(x)$ stretched horizontally away from the y -axis, scaled by a factor of $\frac{1}{c}$.

Reflection of the Graph of $f(x)$ Across the y-Axis

Let f be a function.

- The graph of $f(x) = f(-x)$ is the graph of the $f(x)$ reflected across the y-axis.

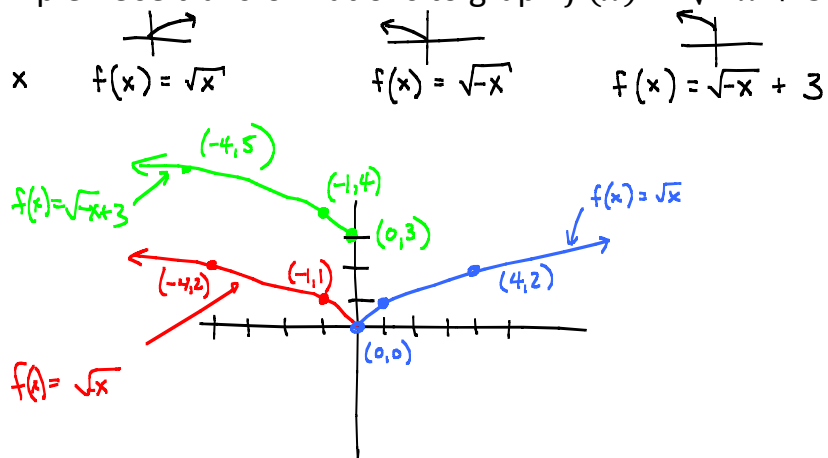
Example: Compare the graph of $f(x) = \sqrt{x}$ to the graph of $f(x) = \sqrt{-x}$.

x	$f(x) = \sqrt{x}$	$f(x) = \sqrt{-x}$

Putting It All Together

Example: Use transformations to graph $f(x) = -\sqrt{x+4}$

Example: Use transformations to graph $f(x) = \sqrt{-x} + 3$



Homework: Section 2.3: 1-45 (odd)