

## Section 2.4: Symmetry and Other Properties of Functions

### Objectives

1. Determine if a function is even, odd, or neither
2. Given a graph, determine intervals in which a function is increasing, decreasing, or constant
3. Determine the average rate of change of a function over an interval

Example of an Even Function:

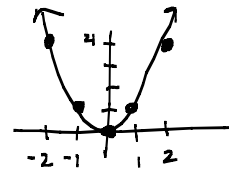
$$f(x) = x^2$$

x	f(x)
-2	4
-1	1
0	0
1	1
2	4

$$f(2) = f(-2)$$

$$f(1) = f(-1)$$

$$f(x) = f(-x) \leftarrow \text{even function}$$



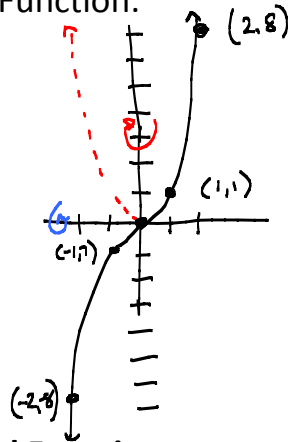
### Definition of an Even Function

A function is symmetric with respect to the y-axis if  $f(x) = f(-x)$  for each  $x$  in the domain of  $f$ . Functions having this property are called even functions.

Example of an Odd Function:

$$f(x) = x^3$$

$x$	$f(x)$
-2	-8
-1	-1
0	0
1	1
2	8



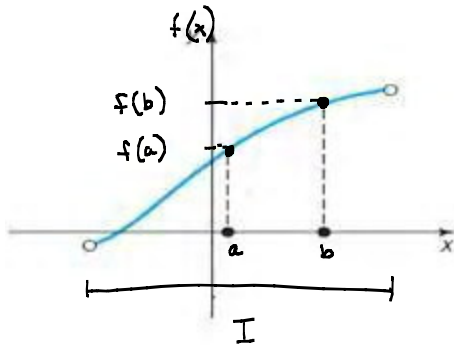
$$f(-x) = -f(x)$$

### Definition of an Odd Function

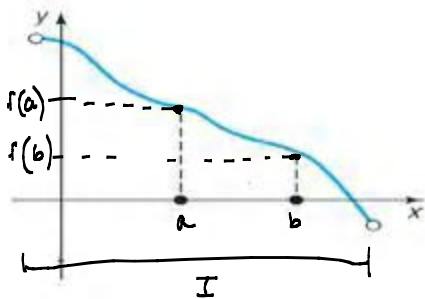
A function is symmetric with respect to the origin if  $f(-x) = -f(x)$  for each  $x$  in the domain of  $f$ . Functions having this property are called odd functions.

## Increasing and Decreasing Functions

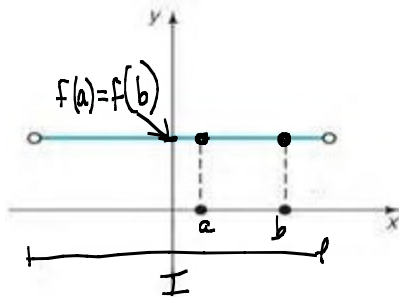
A function  $f$  is increasing on an open interval  $I$  if, for any  $a, b$  in the interval,  $f(a) < f(b)$  for  $a < b$ .



A function  $f$  is decreasing on an open interval  $I$  if, for any  $a, b$  in the interval,  $f(a) > f(b)$  for  $a < b$ .



A function  $f$  is constant on an open interval  $I$  if, for any  $a, b$  in the interval,  $f(a) = f(b)$  for  $a < b$ .



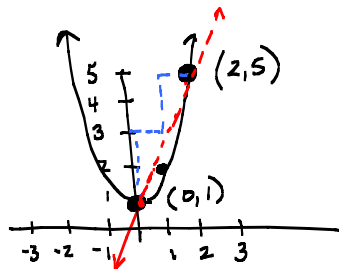
## Find the Average Rate of Change of a Function

The average rate of change of a function  $f$  on an interval  $[x_1, x_2]$  is given by

$$\text{Average rate of change} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Example: Find the average rate of change of  $f(x) = x^2 + 1$  on the following intervals

- a. From 0 to 2



$$\begin{aligned} f(x_2) &= f(2) = 5 \\ f(x_1) &= f(0) = 1 \\ \text{avg rate of change} &= \frac{5-1}{2-0} \\ &= \frac{4}{2} = 2 \end{aligned}$$

- b. From 1 to 3

$$\begin{aligned} f(x_1) &= f(1) = 1^2 + 1 = 2 \\ f(x_2) &= f(3) = 3^2 + 1 = 10 \end{aligned}$$

$$\begin{aligned} \text{avg. rate of change} &= \frac{f(x_2) - f(x_1)}{x_2 - x_1} \\ &= \frac{10 - 2}{3 - 1} = \frac{8}{2} = 4 \end{aligned}$$

Example: Find the average rate of change of  $h(x) = x^2 - 2x + 3$  on the following intervals:

- a. [-1 to 1]

- b. [0, 2]

Section 2.4

**Homework:** 1-7 (odd), 11-31 (all), 45-49 (odd)