

Section 2.5: Equations and Inequalities Involving Absolute Value

Objectives

1. Express the absolute value of a number in terms of the distance on the number line
2. Solve equations involving absolute value
3. Solve inequalities involving absolute value

Absolute Value Equations

If a is a positive real number and if X is any algebraic expression, then

$$\longrightarrow |X| = a \text{ is equivalent to } X = a \text{ or } X = -a.$$

Example: Solve the equation $|x + 2| = 6$.

$$\begin{array}{l} x+2 = 6 \quad \text{or} \quad x+2 = -6 \\ -2 \quad -2 \quad \quad \quad -2 \quad -2 \\ x = 4 \quad \quad \quad \text{or} \quad x = -8 \end{array}$$

Example: Solve the equation $|2x - 4| = 10$.

Example: Solve the equation

Isolate the abs. value by dividing both sides by 3 \rightarrow

$$\begin{array}{l} 3|x+1| = 15 \\ |x+1| = 5 \\ x+1 = -5 \quad \text{or} \quad x+1 = 5 \\ -1 \quad -1 \\ x = -6 \quad \text{or} \quad x = 4 \end{array}$$

Example: Solve the equation $|2x - 4| = -12$.

no solution

Absolute Value Inequalities

If a is a positive real number and if X is any algebraic expression, then

$|X| < a$ is equivalent to $-a < X < a$.

$|X| \leq a$ is equivalent to $-a \leq x \leq a$.

Example: Solve the inequality: $|x| < 3$.

$$-3 < x < 3$$

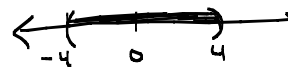
$$(-3, 3)$$

Example: Solve the inequality $|2x| < 8$ and graph the solution set.

$$\frac{-8}{2} < \frac{2x}{2} < \frac{8}{2}$$

$$-4 < x < 4$$

$$(-4, 4)$$



Example: Solve the inequality $|x + 1| < 4$ and graph the solution set.

Example: Solve the inequality $|x + 4| + 3 \leq 5$ and graph the solution set.

Isolate abs.
value first

$$\begin{aligned} & \quad \quad \quad -3 \quad -3 \\ & \quad \quad \quad |x+4| \leq 2 \\ & -2 \leq x+4 \leq 2 \\ & \quad \quad \quad \downarrow \end{aligned}$$

Example: Solve the inequality $|x - 4| < -3$ and graph the solution set.

If a is a positive real number and if X is any algebraic expression, then



$|X| > a$ is equivalent to $X < -a$ or $X > a$

$|X| \geq a$ is equivalent to $X \leq -a$ or $X \geq a$

Example: Solve the inequality $|x| > 3$.

$$x < -3 \quad \text{or} \quad x > 3$$

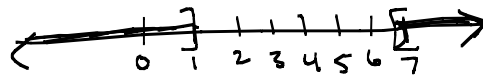
$$(-\infty, -3) \cup (3, \infty)$$

Example: Solve the inequality $|x - 4| \geq 3$ and graph the solution set.

$$\begin{array}{r} x - 4 \leq -3 \\ +4 \quad +4 \end{array} \quad \text{or} \quad \begin{array}{r} x - 4 \geq 3 \\ +4 \quad +4 \end{array}$$

$$x \leq 1 \quad \text{or} \quad x \geq 7$$

$$(-\infty, 1] \cup [7, \infty)$$



Example: Solve the inequality $|x + 4| - 2 > 6$ and graph the solution set.

Homework

Section 2.5: 9-27 (odd), 41-61 (odd)