**Section 3.1**
**Graphs of Quadratic Functions**

**Objectives**

- Graph a Quadratic Function Using Transformations
- Identify the Vertex and Axis of Symmetry of a Quadratic Function

A **quadratic function** is function of the form

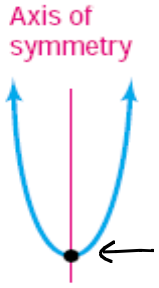
largest exponent is 2.

$$f(x) = ax^2 + bx + c$$

$f(x) = 2x^2 - 3x + 5$ $\quad$ $f(x) = -2x^2 - 3x + 5$

where $a, b$, and $c$ are real numbers and a $\neq 0$.

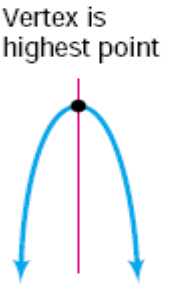
Parabola—the graph of a quadratic function.




Opens up

$a > 0$

↑ coefficient of squared term




Opens down

$a < 0$

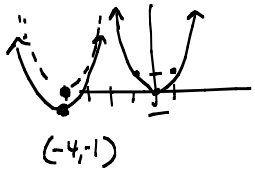
↑

$f(x) = x^2$

$f(x) = (x+4)^2 - 1$

↑ shifts left

$f(x) = 2(x+4)^2 - 1$



Another way to write a quadratic function is

$$f(x) = a(x - h)^2 + k$$

where $(h, k)$ is the vertex of the parabola, and $a$ either stretches (if $|a| > 1$) or compresses (if $0 < |a| < 1$) the parabola.

Also, if $a$ is positive, the parabola opens up. If $a$ is negative, the parabola opens down.

Example: $f(x) = (x - 3)^2 + 1$

vertex : $(3, 1)$ opens up

Example: $f(x) = 2(x - 3)^2 + 1$

vertex : $(3, 1)$

Example: $f(x) = -(x - 3)^2 + 1$

Example: Graph the function $f(x) = x^2 + 6x + 10$. Find the vertex and axis of symmetry.

$f(x) = x^2 + 6x + 10$

$f(x) = x^2 + 6x + 9 + 10 - 9$

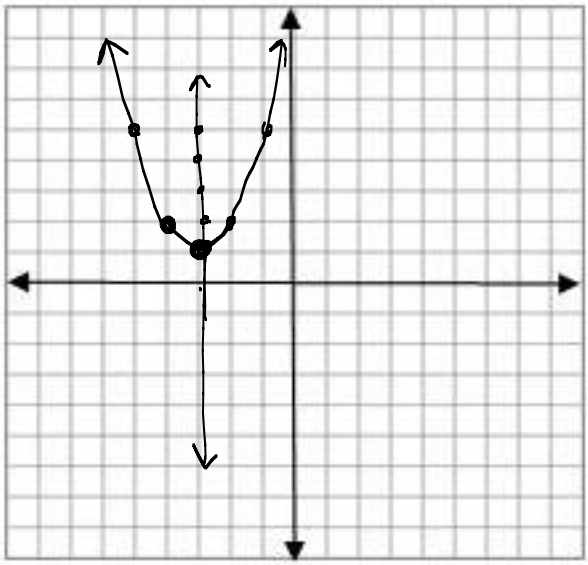
↑ ½ of 6 and square it.

$f(x) = (x+3)(x+3) + 1$

$f(x) = (x+3)^2 + 1$

Vertex: $(-3, 1)$

Axis of symmetry: $x = -3$



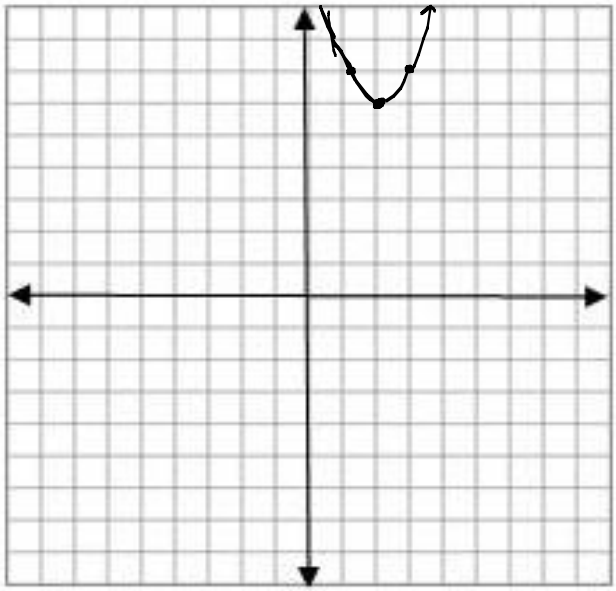
Example: Graph the function $f(x) = x^2 - 4x + 10$. Find the vertex and axis of symmetry.

$f(x) = x^2 - 4x + 4 + 10 - 4$

$= (x-2)^2 + 6$

Vertex = $(2, 6)$

axis of symmetry : $x = 2$



$a = 2$

Example: Graph the function $f(x) = 2x^2 - 4x + 1$. Find the vertex and axis of symmetry.

$f(x) = 2x^2 - 4x + 1$

Factor out leading coefficient from 1st two terms.

$f(x) = 2(x^2 - 2x) + 1$

$f(x) = 2(x^2 - 2x + 1) + 1 - 2$

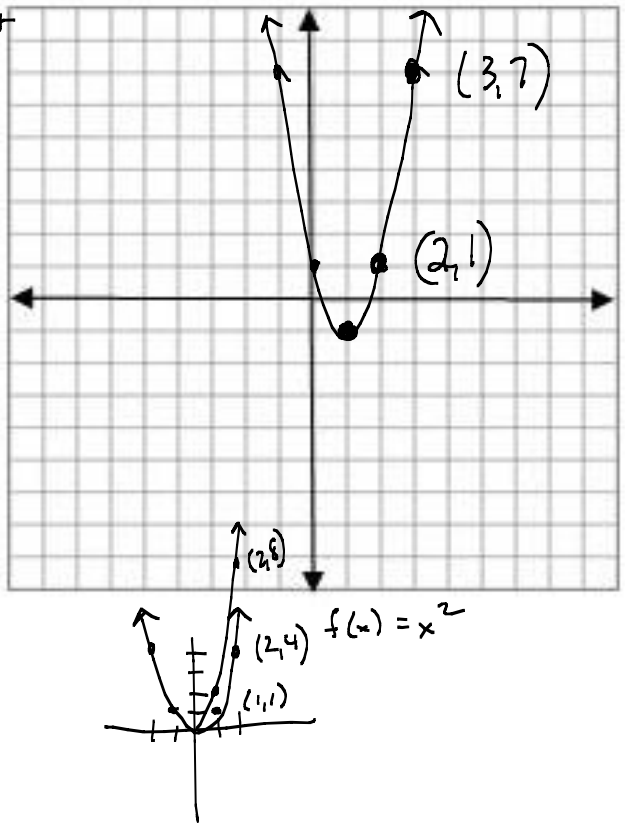
$f(x) = 2(x-1)(x-1) - 1$

$f(x) = 2(x-1)^2 - 1$

stretch ② right ① down ③

| x | f(x) |
|---|---|
| 2 | 1 |
| 3 | 7 |

vertex : $(1, -1)$



**Properties of the Graph of a Quadratic Function**

Given $f(x) = ax^2 + bx + c$, then

1. The $x$-coordinate of the vertex is given by $-\frac{b}{2a}$

2. The $y$-coordinate of the vertex can be found by plugging in the $x$-coordinate (from step 1 above) into $f(x) = ax^2 + bx + c$.

3. Axis of symmetry: the line $x = -\frac{b}{2a}$

Example: Without graphing, locate the vertex and axis of symmetry of the parabola defined by $f(x) = 2x^2 - 4x + 1$.

$a = 2$
$b = -4$
$c = 1$

x-coordinate of vertex: $-\frac{(-4)}{2(2)} = 1$

y-coordinate of vertex: $f(1) = f(1) = 2(1)^2 - 4(1) + 1$

$= 2 - 4 + 1 = -1$

Axis of symmetry: $x = 1$

Example: Without graphing, locate the vertex and axis of symmetry of the parabola defined by $f(x) = -2x^2 + 4x - 6$.

$x = \frac{-b}{2a} = \frac{-4}{2(-2)} = 1$

$f(1) = -2(1)^2 + 4(1) - 6 = -2 + 4 - 6 = -4$

Vertex: $(1, -4)$

axis of symmetry: $x = 1$

**Determining If a Quadratic Function Has a Maximum or Minimum**

Example: Determine if the quadratic function

$$f(x) = x^2 - 6x + 2$$

has a minimum or maximum value. Then find the minimum or maximum value.

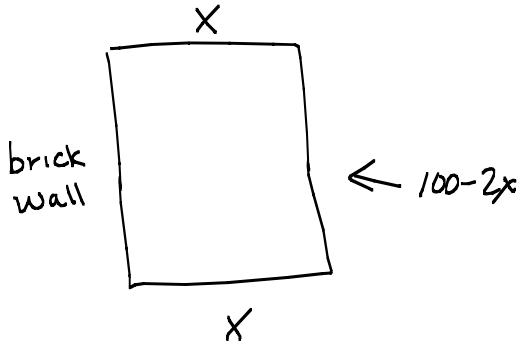
Example: Determine if the quadratic function

$$f(x) = -2x^2 + 8x - 1$$

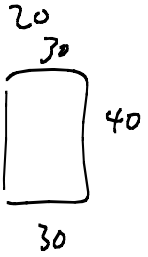
has a minimum or maximum value. Then find the minimum or maximum value.

**Applications**

Example: A rectangular garden plot is to be enclosed with a fence on three of its sides and a brick wall on the fourth side. If 100 feet of fencing material is available, what dimensions will yield the maximum area?



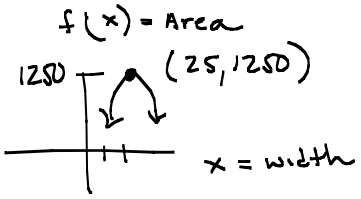





Area = length × width

$f(x) = (100-2x)(x)$

$f(x) = -2x^2 + 100x$




$x = -\frac{b}{2a} = \frac{-100}{2(-2)} = \frac{-100}{-4} = 25$

$f(25) = -2(25)^2 + 100(25)$

$= -2(625) + 2500$

$= -1250 + 2500$

$= 1250$

width = 25

area = 1250

Example: The table below lists the speed at which a car is driven, in miles per hour, and the corresponding gas mileage, in miles per gallon.

| Speed | Gas Mileage |
|---|---|
| 5 | 12 |
| 10 | 17 |
| 25 | 27 |
| 45 | 30 |
| 65 | 25 |
| 75 | 22 |

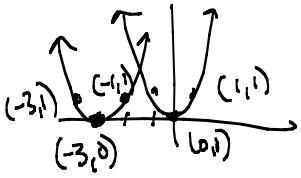
a. Make a scatter plot of the data, using speed as the independent variable x.

b. Find the quadratic function that best fits the given data points.

c. Use the function to predict the gas mileage when the car is driven 35 miles per gallon.

d. Find the speed at which the car's gas mileage is a maximum.

Homework: Section 3.1: 33-57 (odd), 73, 79, 81, ~~83~~ due 11/4




$f(x) = x^2$

$f(x) = (x+3)^2$

| x | f(x) |
|---|---|

| x | f(x) |
|---|---|