Section 3.2
Quadratic Equations

Objectives

- Solve a quadratic equation by factoring, by completing the square, and by using the quadratic formula.

Recall: A quadratic equation is an equation that can be written in the standard form

↙ equation

$$ax^2 + bx + c = 0$$

x = ___

Example: Solve $x^2 - 36 = 0$

$(x+6)(x-6) = 0$

$x+6=0$ or $x-6=0$

$-6\ \ -6$ $\quad$ $+6\ \ +6$

$x = -6$ or $x = 6$

Zero product property: If you multiply two factors together + the product is 0, then either the first factor = 0 or the second factor = 0.

Example: Solve $x^2 - 4x - 21 = 0$

$(x+3)(x-7) = 0$

$x+3=0$ or $x-7=0$

$-3\ \ -3$ $\quad$ $+7\ \ +7$

$x = -3$ or $x = 7$

↓

Example: Solve $-5x^2 + 45x = 0$

$-5x(x-9) = 0$

$-5x = 0$ or $x-9 = 0$

$x = 0$ or $x = 9$

Recall: A quadratic function is a function that can be written in the following form

$$f(x) = ax^2 + bx + c$$

Graphs:

We can find the x-intercepts of this function by setting the function equal to 0.

Example: Find the x-intercepts of the parabola described by the quadratic function.

a. $f(t) = -t^2 + 4t$

b. $f(x) = x^2 + 4x + 4$

c. $f(x) = 6x^2 - x - 2$

Real Zeros—the values of x at which $f(x) = 0$.

Example: Find the real zeros for the previous three problems.

Example Find a possible expression for a quadratic function $f(x)$ having the given zeros:

a. $x = 3$ and $x = 4$

b. $x = -5$ and $x = -3$

c. $x = 0.4$ and $x = 0.8$

Solving a Quadratic Equation by Completing the Square

Example: Solve by completing the square: $x^2 - 6x = 7$

$x^2 - 6x + 9 = 7 + 9$ (x^2 and x terms on left; Constant on right.)

$(x-3)(x-3) = 16$

$\sqrt{(x-3)^2} = \sqrt{16}$

$x - 3 = \pm 4$

$x - 3 = 4$ or $x - 3 = -4$

$x = 7$ or $x = -1$

Example: Solve by completing the square: $x^2 + 8x = 6$

$x^2 + 8x + 16 = 6 + 16$

$(x+4)^2 = 22$

$x + 4 = \pm\sqrt{22}$

$x = -4 + \sqrt{22}$ $\quad x = -4 - \sqrt{22}$

Example: Solve by completing the square: $x^2 - x = 3$

Example: Solve by completing the square: $3x^2 - 6x + 2 = 0$

Factor out the leading coefficient.

$3x^2 - 6x = -2$

$3(x^2 - 2x + 1) = -2 + 3$

$\frac{3(x-1)^2}{3} = \frac{1}{3}$

$(x-1)^2 = \frac{1}{3} \rightarrow x - 1 = \pm\sqrt{\frac{1}{3}}$

$x = 1 \pm \sqrt{\frac{1}{3}} \rightarrow 1 \pm \frac{\sqrt{3}}{3}$

Solving a Quadratic Equation by Using the Quadratic Formula

The solutions to $ax^2 + bx + c = 0$ are given by the following formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example: Solve by using the quadratic formula.

a. $x^2 + x - 5 = 0$

$a = 1$
$b = 1$
$c = -5$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-5)}}{2(1)} = \frac{-1 \pm \sqrt{1 + 20}}{2}$$

$$x = \frac{-1 \pm \sqrt{21}}{2}$$

two solutions

b. $2t^2 + 4t - 5 = 0$

$a = 2$
$b = 4$
$c = -5$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(2)(-5)}}{2(2)}$$

$$= \frac{-4 \pm \sqrt{16 + 40}}{4} = \frac{-4 \pm \sqrt{56}}{4} = \frac{-4 \pm \sqrt{4 \cdot 14}}{4}$$

$$= \frac{-4 \pm 2\sqrt{14}}{4}$$

$$\frac{2(-2 \pm \sqrt{14})}{4_2}$$

$$= \frac{-4}{4} \pm \frac{2\sqrt{14}}{4}$$

$$= -1 \pm \frac{\sqrt{14}}{2}$$

c. $x^2 - 4x + 4 = 0$

$a = 1$
$b = -4$
$c = 4$

$$x = \frac{+4 \pm \sqrt{(-4)^2 - 4(1)(4)}}{2}$$

$$x = \frac{4 \pm \sqrt{16 - 16}}{2} = \frac{4 \pm \sqrt{0}}{2} = \frac{4}{2} = 2$$

one solution

d. $x^2 + 6x + 10 = 0$

$a = 1$

$b = 6$

$c = 10$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(10)}}{2(1)}$$

$$x = \frac{-6 \pm \sqrt{-4}}{2}$$

no real solutions

The Discriminant: $b^2 - 4ac$

If $b^2 - 4ac > 0$, there will be two distinct real solutions.
If $b^2 - 4ac = 0$, there will be one real solution.
If $b^2 - 4ac < 0$, there will be no real solutions.

Example: Use the quadratic formula to solve $x(x - 2) = 2x - 4$. Find the value of the discriminant.

Word Problems

Example: The height of a ball after being dropped from the roof of a 200-foot-tall building is given by $h(t) = -16t^2 + 200$, where t is the time in seconds since the ball was dropped, and $h(t)$ is in feet.

a. When will the ball be 100 feet above the ground?

b. When will the ball reach the ground?

Homework: Section 3.2: 9-69 (every other odd)

19 – 31 (just find x-intercepts)