Section 3.3: Complex Numbers and Quadratic Equations

Objectives:

- Define a complex number
- Perform arithmetic with complex numbers
- Find the complex zeros of a quadratic function
- Find the complex solutions of a quadratic equation

Definition of the Imaginary Number i

The imaginary number i is defined as the number such that $\mathrm{i} = \sqrt{-1}$ and $\mathrm{i}^2 = -1$

Example: Write the number as a pure imaginary number:

$$\sqrt{-64} = \sqrt{(-1)(64)} = \underbrace{\sqrt{-1}}_{i}\sqrt{64} = 8i$$

$$\sqrt{-24} = \sqrt{(-1)(24)} = \sqrt{-1}\sqrt{24} = i\sqrt{24} = i\sqrt{4\cdot 6}$$

perfect square

$$= i\sqrt{4}\sqrt{6}$$

$$= i\cdot 2\cdot\sqrt{6}$$

$$= 2i\sqrt{6}$$

Write i before any $\sqrt{\ }$

$$\sqrt{-\frac{9}{4}} = \sqrt{-1\cdot\frac{9}{4}} = \sqrt{-1}\cdot\sqrt{\frac{9}{4}}$$

$$= i\cdot\frac{3}{2}$$

$$= \frac{3}{2}i$$

Example: Solve the equations:

$x^2 = -25$

$\sqrt{x^2} = \pm\sqrt{-25}$

$x = \pm 5i$

$-x^2 = 12$

$x^2 = -12$

$x = \pm\sqrt{-12}$

$x = \pm\sqrt{-1 \cdot 12} \longrightarrow x = \pm i\sqrt{12} \rightarrow x = \pm i\sqrt{4 \cdot 3}$

$\rightarrow x = \pm i\sqrt{4}\sqrt{3}$

$x = \pm 2i\sqrt{3}$

$5x^2 = -60$

Example: Find all solutions of the quadratic equation using the quadratic formula.

$-x^2 + x - 5 = 0$

$a = -1$

$b = 1$

$c = -5$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-1 \pm \sqrt{1^2 - 4(-1)(-5)}}{2(-1)}$

$x = \frac{-1 \pm \sqrt{1 - 20}}{-2}$

$x = \frac{-1 \pm \sqrt{-19}}{-2}$

$x = \frac{-1 \pm i\sqrt{19}}{-2}$

$-2x^2 + 3x - 1 = 0$

$-7x^2 + 2x - 1 = 0$

Finding the Zeros of a Quadratic Function

x-intercepts

Compute the zeros of the quadratic function:

no x term

$f(x) = 3x^2 + 5$

$0 = 3x^2 + 5$

$a = 3$

$b = 0$

$c = 5$

$$x = \frac{-0 \pm \sqrt{0^2 - 4(3)(5)}}{2(3)}$$

$$x = \frac{\pm\sqrt{-60}}{6}$$

disc. is negative

no real zeros

$$x = \frac{\pm i\sqrt{60}}{6}$$

perfect square

$$x = \frac{\pm i\sqrt{4 \cdot 15}}{6}$$

$$x = \frac{\pm i \cdot \cancel{2}\sqrt{15}}{\cancel{6}\ 3}$$

$$x = \frac{\pm i\sqrt{15}}{3}$$

$f(x) = -3x^2 - 18$

$g(x) = x^2 - x + 1$

$f(x) = 2x^2 - x + 8$

Definition of a Complex Number

A complex number is a number of the form $a + bi$, where a and b are real numbers. a is the real part and b is the imaginary part.

$3 + 5i$ $\qquad$ $2 - 7i$ $\qquad$ $5i$

Addition and Subtraction of Complex Numbers

To add two complex numbers, add their real parts to get the real part of their sum, and add the imaginary parts to get the imaginary part of their sum.

Example: Find $x + y$ and $x - y$

$x = -2i; y = 5 + i$

$-2i + 5 + i = 5 - i$

$x - y = -2i - (5 + i) = -2i - 5 - i = -3i - 5$

parentheses

$x = 2 - 9i; y = -4 + 6i$

$2 - 9i - 4 + 6i = -2 - 3i$

$x = 2 - 7i; y = 11 + 2i$

Multiplication of Complex Numbers

To multiply two complex numbers, apply the rules of multiplication of binomials. (FOIL)

Example: Multiply the following complex numbers

$x = 3 - 2i; y = 5 + i$

$$(3-2i)(5+i) = 15 + \underbrace{3i - 10i} - 2i^2$$

$$= 15 - 7i - 2i^2$$

$i^2 = -1$

Replace i^2 with -1

$$= 15 - 7i - 2(-1)$$

$a + bi$

real imaginary

$$= 15 - 7i + 2$$

$$= 17 - 7i$$

$x = 2 - 7i; y = 11 + 2i$

Division of Complex Numbers

Example: Find the quotient $\frac{x}{y}$ of the following complex numbers:

$x = 1 - 2i; y = 5 + i$

$$\frac{1-2i}{5+i} \cdot \frac{5-i}{5-i}$$

Eliminate the i in the denominator.

conjugate

FOIL

$$= \frac{(1-2i)(5-i)}{(5+i)(5-i)} = \frac{5 - i - 10i + 2i^2}{25 - i^2}$$

FOIL

$$= \frac{5 - 11i + 2(-1)}{25 - (-1)}$$

$= \frac{5 - 11i - 2}{25 + 1}$

$= \frac{3 - 11i}{26}$

$x = 2 - 7i; y = 11 + 2i$

real number in denominator

Homework: Section 3.3: 9-19 (odd), 37-41 (odd), 49-69 (odd)