

## Section 3.4 Quadratic Inequalities

### Objectives

- Solve Quadratic Inequalities Algebraically and Graphically

### Solving Quadratic Inequalities Graphically

Example: Graph the quadratic function  $f(x) = 2x^2 - x - 3$ .  
Then find the solutions of  $x$  for which

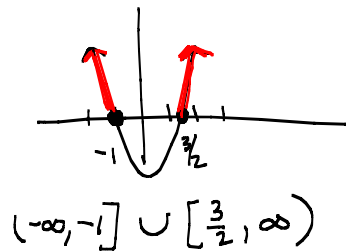
Ⓐ  $f(x) \geq 0$

Ⓑ  $f(x) \leq 0$

Ⓒ  $f(x) < 0$

Ⓓ  $f(x) > 0$   
 $(-\infty, -1) \cup (\frac{3}{2}, \infty)$

a.  $2x^2 - x - 3 \geq 0$



Find the zeros

$$f(x) = 0$$

$$2x^2 - x - 3 = 0$$

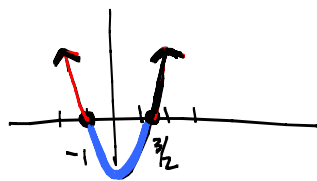
$$(2x-3)(x+1) = 0$$

$$2x-3=0 \quad x+1=0$$

$$+3+3 \quad -1-1$$

$$\cancel{x} = \frac{3}{\cancel{2}} \quad \text{or} \quad x = -1$$

b.  $2x^2 - x - 3 \leq 0$   
 $0 \leq 0$

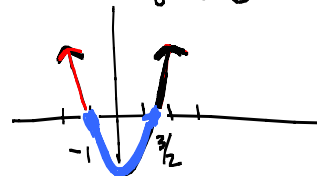


↑  
include endpoint  
in solution set  
because of  $\leq$

c.  $2x^2 - x - 3 < 0$

$$2(-1)^2 - (-1) - 3$$

$$0 < 0$$



↑  
don't include  
end point  
because of  $<$

Example: Solve  $x^2 - 3x - 10 > 0$  graphically. ↙ above

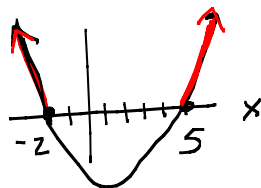
① Find zeros (x-intercepts)

$$x^2 - 3x - 10 = 0$$

$$(x-5)(x+2) = 0$$

$$x=5 \text{ or } x=-2$$

② Graph



③ State solution set in interval notation.

$$(-\infty, -2) \cup (5, \infty)$$

Is -2 in the solution set?  $(-2)^2 - 3(-2) - 10 > 0$

$$4 + 6 - 10 > 0$$

$$0 > 0$$

Example: Solve  $x^2 + 2x - 3 < 0$  graphically.

no

## Steps for Solving Quadratic Inequalities Algebraically

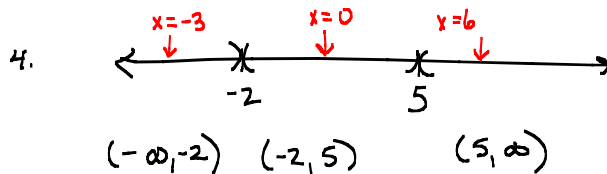
1. Write the inequality so one side is zero.
2. Factor the expression on the nonzero side.
3. Find the zeros of the nonzero side.
4. Using those zeros, break the number line into intervals.
5. Pick a test point in each interval and determine if the function is positive or negative at that test point

Example: Solve  $x^2 - 3x - 10 > 0$  algebraically.

1. ✓

2,3  $(x-5)(x+2) = 0$

$$\begin{array}{ccc} x-5=0 & \text{or} & x+2=0 \\ +5 & & -2 \\ x=5 & \text{or} & x=-2 \end{array}$$



Does  $x = -3$  work?  
 $(-3)^2 - 3(-3) - 10 > 0$  ✓  
 Does  $x = 0$  work?  
 $0^2 - 3(0) - 10 > 0$  ✗  
 Does  $x = 6$  work?  
 $6^2 - 3(6) - 10 > 0$  ✓

5. Test points in red.

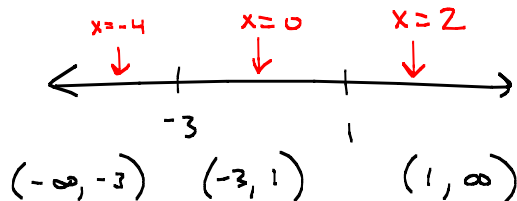
Solution:  $(-\infty, -2) \cup (5, \infty)$

Example: Solve  $x^2 + 2x - 3 < 0$  algebraically.

$$x^2 + 2x - 3 = 0$$

$$(x+3)(x-1) = 0$$

$$x = -3, \quad x = 1$$



Check test points

$$\begin{array}{l} (-4)^2 + 2(-4) - 3 < 0 \quad \text{no} \\ (0)^2 + 2(0) - 3 < 0 \quad \text{✓} \\ (2)^2 + 2(0) - 3 < 0 \quad \text{no} \end{array}$$

Solution:  $(-3, 1)$

Example: Solve  $x^2 + 7x + 12 \leq 0$  algebraically.

Example: Solve  $x^2 + 8x \geq 0$  algebraically.

Example: Solve  $x^2 - 1 \leq 0$  algebraically.

Factor  $(x+1)(x-1) = 0$


Example: Solve  $6x^2 \geq 13x - 5$  algebraically.

①  $6x^2 - 13x + 5 \geq 0$

move everything to one side

solution set will have  
square brackets

Example: Solve  $-3x^2 \leq -7x - 6$  algebraically.

  
Example: Solve  $6(x^2 - 1) \geq 5x$  algebraically and graphically.

$$6x^2 - 6 \geq 5x$$

$$6x^2 - 5x - 6 \geq 0$$

## Solving Quadratic Inequalities That Are Not Factorable

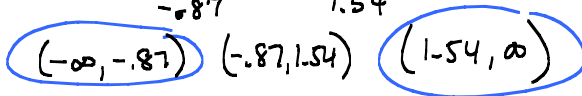
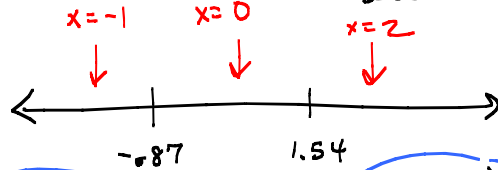
Example: Solve  $3x^2 - 2x \geq 4$ .

1. Bring all terms to one side.

$$3x^2 - 2x - 4 \geq 0$$

2.  $3x^2 - 2x - 4 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-4)}}{2(3)}$$



Does -1 work?  
 $3(-1)^2 - 2(-1) - 4 \geq 0$  ✓

Does 0 work?  
 $3(0)^2 - 2(0) - 4 \geq 0$  no

2 does work.

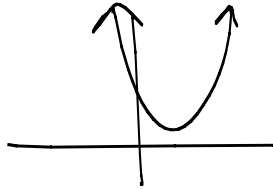
$$(-\infty, -0.87] \cup [1.54, \infty)$$

Example: Solve  $2x^2 + 3x \leq 1$ .

## Solving Quadratic Inequalities With No Real Solution

Example: Solve  $x^2 + x + 1 < 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1^2 - 4(1)(1)}}{2(1)}$$



$$= \frac{-1 \pm \sqrt{1-4}}{2}$$

$$= \frac{-1 \pm \sqrt{-3}}{2} \leftarrow \text{discrim}$$

$$= \frac{-1 \pm i\sqrt{3}}{2} \leftarrow \text{complex zeros}$$

no real solutions

Example: Solve  $-2x^2 + x - 1 < 0$

Homework: Section 3.4: 1-8 (all), 15-37 (odd)