Section 4.1
Polynomial Functions and Models

Objectives

- Identify Polynomial Functions and Their Degree
- Identify the Zeros of a Polynomial Function and Their Multiplicity
- Analyze the Graph of a Polynomial Function

Examples of polynomial functions:

$f(x) = 7x^6 - 3x^4 + x + 5$ ←

$f(x) = x^2 - 4x + 1$ ← quadratic function

$f(x) = x + 1$ ← linear function

A **polynomial function** is a function of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

(a_n, a_{n-1}, ... a_1, a_0: coefficients; x: variable)

where $a_n, a_{n-1}, \ldots a_1, a_0$ are real numbers and n is a nonnegative integer. The domain is the set of real numbers.

↑ you're allow to substitute any real number for x.

Examples of functions that are not polynomials:

$f(x) = \sqrt{x^3 + 1}$

$f(x) = \frac{1}{x^6} + \frac{1}{x^4} - \frac{3}{x^2}$

Degree of a polynomial—the largest power of x that appears.

Example: Find the degree of the following polynomial functions:

$f(x) = 5x^4 + 3x^3$ degree = 4

$g(x) = 4x^7 + 3x^6 - 2x^3$ degree = 7

$f(x) = 9x$ degree = 1

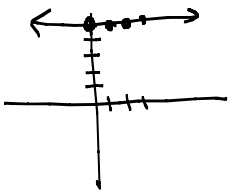
line

slope = 9

y-int = 0

$g(x) = 5$ degree = 0

x	g(x)
0	5
1	5
2	5



$h(x) = x(x + 1)$

$h(x) = x^2 + x$ degree = 2

$f(x) = (x + 2)(x + 5)(x - 7)$ degree = 3

$f(x) = 2(x-1)^2(x+1)$ degree = 3

$2(x-1)(x-1)(x+1)$

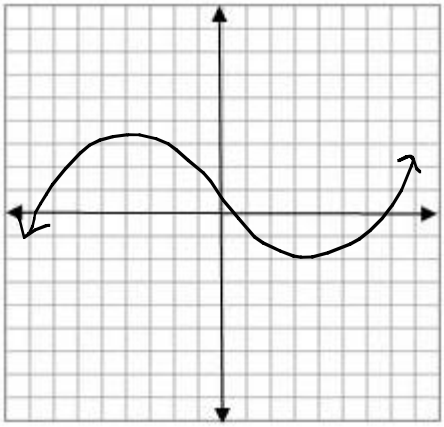
$2x^3 + \ldots\ldots$

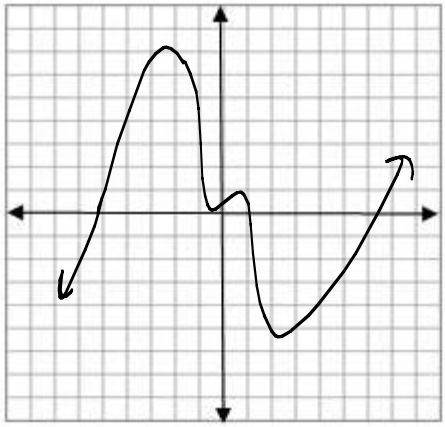
$f(x) = -3x^2(x+2)^3$ degree = 5

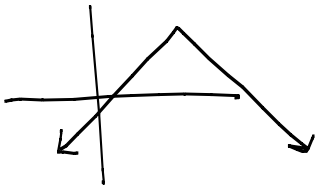
$-3x^2(x+2)(x+2)(x+2)$

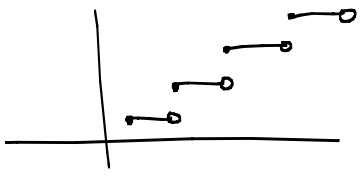
$-3x^5 + \ldots\ldots$

The graph of every polynomial function is both smooth and continuous.









Power Functions

A **power function** of degree n is a function of the form

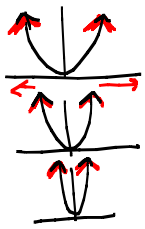
$$f(x) = ax^n$$

where a is a real number, $a \neq 0$, and $n > 0$ is an integer.

Examples: $f(x) = 3x^4$, $f(x) = -4x^9$

a is positive, n is even:

$(a = 1)$ $f(x) = x^2$, $f(x) = x^4$, $f(x) = x^6$



end behavior

As $x \to \infty$, $f(x) \to \infty$

As $x \to -\infty$, $f(x) \to \infty$

a is negative, n is even:

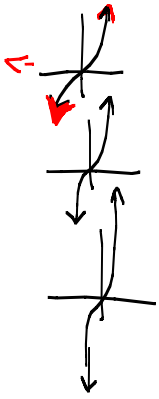
$(a = -1)$ $f(x) = -x^2$, $f(x) = -x^4$, $f(x) = -x^6$

As $x \to \infty$, $f(x) \to -\infty$

As $x \to -\infty$, $f(x) \to -\infty$

a is positive, n is odd:

$a = 1$ $f(x) = x^3$, $f(x) = x^5$, $f(x) = x^7$ (exponent)



As $x \to \infty$, $f(x) \to \infty$

As $x \to -\infty$, $f(x) \to -\infty$

a is negative, n is odd:

As $x \to \infty$, $f(x) \to -\infty$

As $x \to -\infty$, $f(x) \to +\infty$

End Behavior

For large values of x, either positive or negative, the graph of the polynomial

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

resembles the graph of the power function

$$y = a_n x^n$$

Example: Determine the end behavior of the function $f(x) = 10x^6 - 23x^3 + 4x - 2$

power function : $f(x) = 10x^6$ ← even, ↑ positive

As $x \to \infty$, $f(x) \to \infty$

As $x \to -\infty$, $f(x) \to \infty$

Example: Determine the end behavior of the function $f(x) = -5x^8 - 3x^3 + 2x^2$

even, neg.

Example: Determine the end behavior of the function $f(x) = 2x^3 + 4x - 1$

Identify the Zeros (X-intercepts) of a Polynomial Function by Factoring

Example: Find the zeros of the function $f(x) = x^2 - 6x + 8$

Set $f(x) = 0$

$0 = x^2 - 6x + 8$

$0 = (x-2)(x-4)$

$x-2=0$ $\quad$ $x-4=0$

$+2 \quad +2$

$x = 2$ $\quad$ $x = 4$

Example: Find the zeros of the function $f(x) = 3x^3 - 27x$

$0 = 3x^3 - 27x$

$0 = 3x(x^2 - 9)$

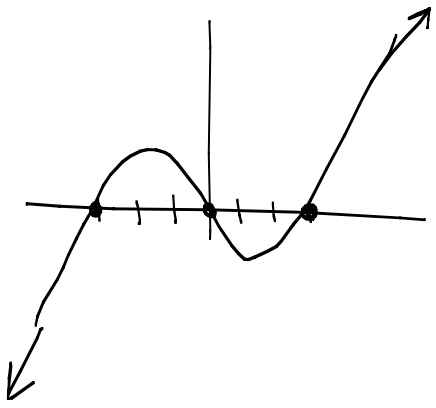
$0 = 3x(x+3)(x-3)$

$3x = 0$ or $x+3=0$ or $x-3=0$

$x = 0$ or $x = -3$ or $x = 3$

power function: $f(x) = 3x^3$ ← odd

↑ pos

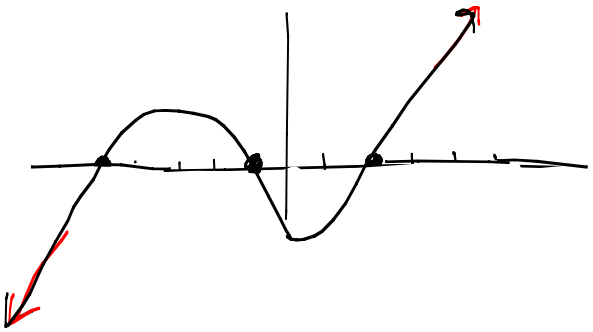


Example: Find the zeros of the function $f(x) = (x + 1)(x - 2)(x + 5)$

Zeros: $0 = (x+1)(x-2)(x+5)$

$x+1=0$ or $x-2=0$ or $x+5=0$

$x=-1$ $x=2$ $x=-5$



power function:

$f(x) = x^3$

Analyzing the Graph of a Polynomial Function

Example: Analyze the graph of $f(x) = x^3 - 2x^2 - 3x$

Step 1: What is the end behavior?

Step 2: Find the y-intercept.

Step 3: Find the x-intercepts.

Step 4: Determine the signs of function values.

Step 5: Sketch the graph.

Example: Analyze the graph of $f(x) = x^2(x+2)(x-2)$.

Step 1: What is the end behavior?

Step 2: Find the y-intercept.

Step 3: Find the x-intercepts.

Step 4: Determine the signs of function values.

Step 5: Sketch the graph.

Homework: Section 4.1: 11-31 (odd), 49-63 (odd)