

## Section 4.2

### More on Graphs of Polynomial Functions and Models

#### Objectives

- Define the multiplicity of a polynomial
- Check for symmetry of polynomial functions
- Sketch a complete graph

#### Multiplicity

Sometimes when you factor a polynomial, the same factor occurs more than once. The multiplicity of a zero refers to the exponent on the corresponding factor.

Example:  $f(x) = (x - 6)^3(x + 2)^2$

Example:  $f(x) = t^3 + 2t^2 + t$

If the multiplicity of a real zero is even, the graph of the function touches the x-axis at the corresponding x-intercept.

If the multiplicity of a real zero is odd, the graph of the function crosses the x-axis at the corresponding x-intercept.

Example: For the function  $f(x) = 4(x + 4)(x + 3)^2$ , list each real zero and its multiplicity. Also, determine whether the graph crosses or touches the x-axis at each x-intercept.

Example: For the function  $f(x) = 2(x - 3)(x + 4)^3$ , list each real zero and its multiplicity. Also, determine whether the graph crosses or touches the x-axis at each x-intercept.

## Checking for Symmetry

Recall: Even vs. Odd functions

Even function—symmetric about the y-axis:  $f(-x) = f(x)$

Odd function—symmetric about the origin:  $-f(-x) = f(x)$

Example: Decide whether the function is even, odd or neither:  $f(x) = -x^4 + x$

## Finding Local Extrema and Sketching a Complete Graph

Local Extrema (Turning Points)—points on the graph where the graph changes from an increasing function to a decreasing function, or vice versa.

Local Maxima— where a graph “peaks”

Local Minima—where a graph “dips”

We will use the graphing calculator to find the local extrema.

Example: Find the local extrema of  $f(x) = x^5 - x^3$

## Analyzing the Graph of a Polynomial Function

Example: Analyze the graph of  $f(x) = -2x^4 + 8x^2$

Step 1: What is the end behavior?

Step 2: Find the y-intercept.

Step 3: Find the x-intercepts. Determine whether the graph crosses or touches the  $x$ -axis at each  $x$ -intercept.

Step 4: Check for symmetry.

Step 5: Make a table of test values to determine the sign of  $f$ .

Step 6: Draw a complete graph by hand.



Example: Analyze the graph of  $f(x) = x^2(x - 3)(x^2 + 3)$ .

Step 1: What is the end behavior?

Step 2: Find the y-intercept.

Step 3: Find the x-intercepts. Determine whether the graph crosses or touches the x-axis at each x-intercept.

Step 4: Check for symmetry.

Step 5: Make a table of test values to determine the sign of f.

Step 6: Draw a complete graph by hand.

### **Finding a Polynomial Given its Zeros**

Example: Find a polynomial of degree 3 with zeros 1, 2, and 3, each of multiplicity 1.

Example: Find a polynomial of degree 3 with zeros -1, 4, 0, and 2, each of multiplicity 1.

Example: Find a polynomial of degree 4 that has zeros at -1 and 2, each of multiplicity 1, and a zero at -2 of multiplicity 2.

Example: Find a polynomial of degree 5 that has zeros at -5, 3 and 2, each of multiplicity 1, and a zero at 1 of multiplicity 2.

## **Homework**

Section 4.2: 27-45 (odd)