Section P.3
nth Roots; Rational Exponents

Objectives

1. Work with nth Roots
2. Simplify Radicals
3. Rationalize Denominators
4. [illegible]
5. Simplify Expressions with Rational Exponents

$\sqrt[4]{16} = 2$

$\sqrt[3]{27} = 3$

$\sqrt[3]{8} = 2$

$\sqrt[3]{-8} = -2$

The nth root of a number a is symbolized by $\sqrt[n]{a}$.

$\sqrt[n]{a} = b$ means $a = b^n$

The symbol $\sqrt[n]{a}$ is called the radical, the integer *n* is called the index, *a* is called the radicand.

Perfect Roots—expressions that can simplify into rational numbers.

Perfect squares: $16, 4, x^2, x^4, x^6$

Perfect cubes: $8, 27, x^3, x^6$

Example: $\sqrt[3]{27}$

Example: $\sqrt[5]{-32}$

Example: $\sqrt[4]{\frac{1}{16}}$

Example: $\sqrt[4]{(-3)^4}$

Example: $\sqrt[2]{x^8} = x^4$

Example: $\sqrt[3]{y^6} = y^2$

$$\sqrt[3]{x^3 y^6 z^{12}} = x y^2 z^4$$

$$\sqrt[3]{x^3 + y^6 + z^{12}}$$

Simplifying Radicals

$\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$ *"The nth root of a product is the product of nth roots."*

$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ *"The nth root of a quotient is the quotient of nth roots."*

To simplify radials, remove from the radicals any perfect roots that occur as factors.

Example: $\sqrt{24} = \sqrt{\underset{\substack{\uparrow \\ \text{perfect square}}}{\underline{4}} \cdot \underline{6}} = \sqrt{4} \cdot \sqrt{6} = \underset{\substack{\uparrow \\ \text{multiply}}}{2\sqrt{6}}$

Example: $\sqrt{72}$

Example: $\sqrt[3]{16} = \sqrt[3]{\underset{\substack{\uparrow \\ \text{perfect cube}}}{\underline{8}} \cdot \underline{2}} = \sqrt[3]{8} \cdot \sqrt[3]{2}$

$= 2\sqrt[3]{2}$

Example: $\sqrt{20x^5} = \sqrt{\underbrace{4x^4}_{\text{perfect square}} \cdot 5x} = \sqrt{4x^4}\sqrt{5x} = 2x^2\sqrt{5x}$

even

Example: $\sqrt{98y^3z^8} = \sqrt{\underbrace{49y^2z^8}_{\text{perfect square}} \cdot 2y} = \sqrt{49y^2z^8}\sqrt{2y}$

$= 7yz^4\sqrt{2y}$

Example: $\sqrt[3]{-24x^4y^6} = \sqrt[3]{\underbrace{-8x^3y^6}_{\text{perfect cube}} \cdot 3x} = \sqrt[3]{-8x^3y^6} \cdot \sqrt[3]{3x}$

$= -2xy^2\sqrt[3]{3x}$

Multiplying Radicals

Example: $\sqrt{5x}\sqrt{20x^3} = \sqrt{5x \cdot 20x^3} = \sqrt{5 \cdot 20 \cdot x \cdot x^3} = \sqrt{100x^4}$

$= 10x^2$

Example: $\left(3\sqrt{6}\right)\left(2\sqrt{2}\right) = 3 \cdot 2 \cdot \sqrt{6} \cdot \sqrt{2} = 6\sqrt{12}$

$= 6\sqrt{\underline{4} \cdot \underline{3}}$

perfect square

$= 6 \cdot \sqrt{4} \cdot \sqrt{3} = 6 \cdot 2\sqrt{3} = 12\sqrt{3}$

Example: $\sqrt[3]{\frac{3xy^2}{81x^4y^2}}$

Combining Like Radicals

Example: $3\sqrt{\underline{5}} + 4\sqrt{\underline{5}} = 7\sqrt{5}$

Example: $8\sqrt{27} + 4\sqrt{3}$

$\_\_\sqrt{3} + 4\sqrt{3}$

Example: $(\sqrt{5}-2)(\sqrt{5}+2)$

Rationalize Denominators

Rationalize the denominator means "write without roots."

Example: Rationalize the denominator: $\frac{1}{\sqrt{3}} \cdot \underbrace{\frac{\sqrt{3}}{\sqrt{3}}}_{1} = \frac{\sqrt{3}}{3}$

Example: Rationalize the denominator: $\frac{x}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{x\sqrt{5}}{5}$

Example: Rationalize the denominator: $\frac{1}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{4}}{\sqrt[3]{4}} = \frac{\sqrt[3]{4}}{\sqrt[3]{8}} = \frac{\sqrt[3]{4}}{2}$

goal: perfect cube in denominator

Example: Rationalize the denominator: $\frac{1}{\sqrt[3]{5}} \cdot \frac{\sqrt[3]{25}}{\sqrt[3]{25}} = \frac{\sqrt[3]{25}}{\sqrt[3]{125}}$

perfect cube

Example: Rationalize the denominator: $\frac{\sqrt{2}}{\sqrt{7}+2}$

Read

Example: Rationalize the denominator: $\frac{3\sqrt{5}}{\sqrt{3}-1}$

Simplifying Expressions with Rational Exponents

$$a^{1/n} = \sqrt[n]{a}$$

nth root of a

Examples:

$25^{1/2} = \sqrt[2]{25}$

$36^{1/2} = \sqrt{36}$

$8^{1/3} = \sqrt[3]{8} =$

raise to mth power

nth root

$$a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$$

$25^{3/2} = \left(\sqrt[2]{25}\right)^3$

Examples:

power root

Example: Simplify $4^{3/2}$ $= \left(\sqrt[2]{4}\right)^3 = 2^3 = 8$

power root

Example: Simplify $16^{3/4}$ $= \left(\sqrt[4]{16}\right)^3 = 2^3 = 8$

Example: Simplify $\left(\frac{64}{27}\right)^{2/3}$

When the exponent is a negative fraction, take the reciprocal of the base and raise to a positive exponent.

Example: Simplify $16^{-3/4}$ $= \frac{1}{16^{3/4}} = \frac{1}{8}$

Example: Simplify $25^{-3/2}$ $= \frac{1}{25^{3/2}} = \frac{1}{(\sqrt{25})^3} = \frac{1}{5^3} = \frac{1}{125}$

Example: Simplify $\left(\frac{27}{8}\right)^{-2/3}$

Example: Simplify $3^{2/3} \cdot 3^{1/3}$ $x^2 \cdot x^3 = x^5.$

Example: Simplify $\dfrac{2^{1/2}}{2^{1/4}}$

Homework: Section P.3: 1 – 65 (every other odd) 1, 5, 9, ...