

Section P.4 Polynomials

Objectives

1. Define a polynomial
2. Write a polynomial in descending order
3. Add and subtract polynomials
4. Multiply polynomials

$$3x^2 - 2x + 5$$

$$\underbrace{9x^7} + \underbrace{4x^3} - \underbrace{2x}$$

↑ ↑ ↑

A polynomial in one variable is an algebraic expression of the form:

$$\longrightarrow a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

where $a_n, a_{n-1}, \dots, a_1, a_0$ are the coefficients of the polynomial, $n \geq 0$ is an integer, an x is a variable.

Example:

Polynomial Terminology

The **degree** of a polynomial is the highest power to which a variable is raised.

$$x^7 - 2x^4 + 3 \quad \text{degree } 7$$

The parts of a polynomial separated by plus signs are called **terms**.

$$x^7 + -2x^4 + 3 \quad x^7, -2x^4, 3 \quad 3 \text{ terms}$$

The numbers a_n, a_{n-1}, \dots, a_0 are called **coefficients**.

$$1, -2, 3$$

The **constant term** is a_0 .

Example: Write the following polynomial in descending order and find its degree, terms coefficients and constant term.

$$\begin{array}{c} \underline{3y^2} + \underline{12y^3} + 6 \\ 12y^3 + 3y^2 + 6 \end{array}$$

Monomial: A polynomial with 1 term
 $x^4, 36x^5$

Binomial: A polynomial with 2 terms
 $x+5, y^2-1$

Trinomial: A polynomial with 3 terms
 $x^2+3x-1, 2x^5-17x+15$

Addition and Subtraction of Polynomials

Like terms—Two monomials having the same degree and the same variable.

We can add and subtract polynomials by combining like terms.

Example: $2x^2 + 4x^2 = 6x^2$
 $\uparrow \quad \uparrow$

$$5x + 7x = 12x$$

$$15x^7 + 2x^7 = 17x^7$$

$$\underline{15a} - \underline{12a} = 3a$$

Example: Add $2x^4 + 5x^3$ and $4x^3 - 10x^2$.

$$2x^4 + \underline{5x^3} + \underline{4x^3} - 10x^2 = 2x^4 + 9x^3 - 10x^2$$

Example: $(3y^5 + 12y^3 + 6) + (9y^4 + 10y^3 - 7)$

$$3y^5 + 12y^3 + 6 + 9y^4 + 10y^3 - 7 = 3y^5 + 9y^4 + 22y^3 - 1$$

Example: $(13y^5 + 6y^3 + 3) - (14y^4 - 10y^3 - 7)$

$$\begin{aligned} & 13y^5 + 6y^3 + 3 - 14y^4 + 10y^3 + 7 \\ & = 13y^5 - 14y^4 + 16y^3 + 10 \end{aligned}$$

Example: $(7x^4 - 6) - (7x^7 + 5x^4 + 17)$

Multiplying Polynomials

Example: Multiply $(\underline{3x^4})(\underline{-5x^9}) = -15x^{4+9} = -15x^{13}$

Example: Multiply $\underline{2}(x^2 + 6x - 1)$. $= 2x^2 + 12x - 2$

distributive property

Example: Multiply $\underline{5y}(y^2 - 6y)$. $= \underline{5y} \cdot y^2 - \underline{5y} \cdot 6y = 5y^3 - 30y^2$

distributive property

Example: Multiply $(\underline{x+2})(\underline{x+3})$. $= x \cdot x + 3 \cdot x + 2 \cdot x + 2 \cdot 3$

FOIL - Use for
multiplying two
binomials

F O I L

$$= x^2 + 3x + 2x + 6$$

$$= x^2 + 5x + 6$$

$(\underline{x+3})(\underline{x-3}) = x^2 - \cancel{3x} + \cancel{3x} - 9 = x^2 - 9$

FOIL

Example: Multiply $(2x^2 - 7)(4x + 1)$.

Example: Multiply $(5x - 7)(x^2 + 3x - 1)$.

$$\begin{aligned} & 5x \cdot x^2 + 5x \cdot 3x + 5x(-1) - 7x^2 - 21x + 7 \\ &= 5x^3 + \underline{15x^2} - \underline{5x} - \underline{7x^2} - \underline{21x} + 7 \\ &= 5x^3 + 8x^2 - 26x + 7 \end{aligned}$$

Homework: Section P.4: 1-78 (every other odd)

30. $20u^3 - 50u$

36. $x^2 + 3x - 28$

46. $t^2 - 10t + 25$

58. $z^2 - 49$