

Section P.5 Factoring

Objectives

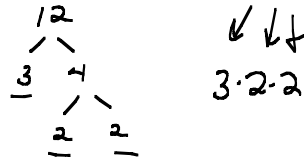
1. Factoring the greatest common factor
2. Factor by grouping
3. Factor trinomials
4. Factor differences of squares and perfect square trinomials

Factoring

What does it mean to factor a number?

$$10 = 2 \cdot 5$$

$$12 = 3 \cdot 4 \\ 2 \cdot 6$$



$$20 =$$

Factoring the Greatest Common Factor (GCF)

Example: Factor $\underline{6}x + \underline{2}$. = $\underline{2} \cdot 3 \cdot x + \underline{2} = 2(3x+1)$

Example: Factor $3x^2 + 9x. = 3x \cdot x + 3x \cdot 3$

$$\rightarrow = 3x(x+3)$$

Example: Factor $20x^5 - 30x. = 10x \cdot 2x^4 - 10x \cdot 3$

$$5x(4x^4 - 6) = 10x(2x^4 - 3)$$

Example: Factor $5(3x - 7) + x(3x - 7). \quad \underline{GCF = 3x - 7}$

$$= (3x - 7)(5 + x)$$

Factor: $3(\cancel{x+1}) + x(\cancel{x+1}) \quad GCF = x+1$
 $(x+1)(3+x)$

Example: Factor $(x-1)^2 - 2(x-1). \quad \overset{x=2}{\boxed{(x-1)^2 - 2(x-1)}}$ $x^2 - 2x + 1 - 2x + 2$
 $= \underline{x^2 - 4x + 3}$

$$= (\cancel{x-1})(\cancel{x-1}) - 2(\cancel{x-1})$$

$$= (x-1)(x-1-2)$$

$$= \boxed{(x-1)(x-3)} \quad \leftarrow \overset{FOIL}{=} \underline{x^2 - 4x + 3}$$

Factoring by Grouping \leftarrow 4 terms

Example: $x^3 + 3x^2 + 2x + 6.$

$$= x^2(x+3) + 2(x+3)$$
$$= (x+3)(x^2+2)$$

- ① Group first two and last two terms together
- ② Pull out the GCF in each group

Example: Factor $x^3 + 2x^2 - x - 2.$

$$= x^2(x+2) - 1(x+2)$$
$$= (x+2)(x^2-1) \leftarrow$$
$$= (x+2)(\quad)(\quad) \downarrow$$

Example: Factor $x^3 + 3x^2 + x + 3.$ \leftarrow

$$= x^2(x+3) + 1(x+3)$$
$$= (x+3)(x^2+1)$$

OR reorder

$$x^3 + x + 3x^2 + 3$$
$$= x(x^2+1) + 3(x^2+1) \downarrow$$
$$= (x^2+1)(x+3)$$

Factoring a Difference of Squares

$$x^2 - y^2 = (x + y)(x - y)$$
$$x^2 - 1^2 = (x + 1)(x - 1)$$

Example: $x^2 - 25 = (x + 5)(x - 5)$

$$4x^2 - 25 = (2x + 5)(2x - 5)$$

Example: $9 - 16y^2$

Example: $x^4 - 25 = (x^2 + 5)(x^2 - 5)$

Factoring Trinomials

Example: Factor $x^2 + 7x + 12 = (x + \underline{3})(x + \underline{4})$

↑
Coefficient
of x^2 term
is 1.

What two numbers
multiply to 12 and
add to 7?

check by FOILing

$$\underline{x^2 + 9x + 20 = (x + 4)(x + 5)}$$

Example: Factor $x^2 + 2x - 8 = (x + 4)(x - 2)$

Mult. to -8 and add to +2

+4, -2

Factor out
GCF first!

Example: Factor $3x^2 + 6x - 9 = 3(x^2 + 2x - 3)$
 $= 3(x + 3)(x - 1)$

Mult to -3 and add
to 2.

3, -1

Example: Factor $ax^2 + bx + c$
 $6x^2 + 11x - 7$.

$$\begin{aligned}
 & \begin{array}{c} \swarrow \quad \searrow \\ (6x^2 + 14x - 3x - 7) \\ = 2x(3x+7) - 1(3x+7) \\ = (3x+7)(2x-1) \end{array}
 \end{aligned}$$

① Identify $a = 6$
 $b = 11$
 $c = -7$

② Compute $ac = -42$

③ Find two numbers that multiply to $ac = -42$ and add to $b = 11$

$14, -3$
 ④ Break apart the middle term using those numbers

⑤ Factor by grouping

Example: Factor $3x^2 - 10x - 25$.

$$ac = -75$$

$$b = -10$$

Two numbers $-15, 5$

$$3x^2 - 15x + 5x - 25$$

$$= 3x(x-5) + 5(x-5)$$

$$= (x-5)(3x+5) \quad \leftarrow 39+40$$

Homework: Section P.5: 7 - 33 (every other odd), 61 - 109 (every other odd)

