Practice Test 4 Solutions

1. $(-\infty,-2] \cup [0,\infty)$

2. $f(t) \geq g(t)$ means the intervals over which the values of $f(t)$ are greater than or equal to $g(t)$ (i.e. the graph of $f(t)$ is "higher" than in graph of $g(t)$.)

$(-\infty,-4] \cup [0,\infty)$

3. $x^2-2x-8<0$

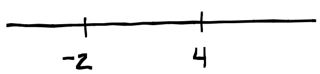
Find zeros of $f(x)=x^2-2x-8$

$x^2-2x-8=0$

$(x-4)(x+2)=0$

$x=4,\ x=-2$

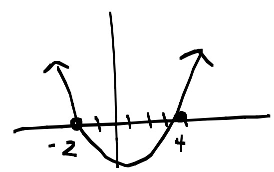
Algebraically



Intervals	Test point	Function value $f(x)=x^2-2x-8$	+/-
$(-\infty,-2)$	-3	7	+
$(-2,4)$	0	-8	-
$(4,\infty)$	5	7	+

Solution: $(-2,4)$

Graphically



Solution: $(-2,4)$

4. $x^2+16x+64 \geq 0$

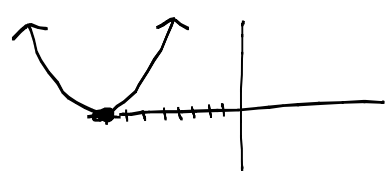
Find zeros of $f(x)=x^2+16x+64$

$x^2+16x+64=0$

$(x+8)(x+8)=0$

$x=-8$

Graphically



$f(x) \geq 0$ for all real numbers. Therefore the solution is $(-\infty,\infty)$.

5. $-x^2+18x-81<0$

Find zeros of $f(x)=-x^2+18x-81<0$

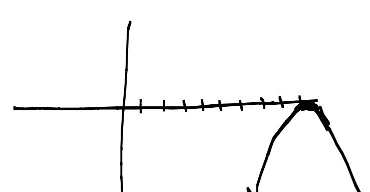
$-x^2+18x-81=0$

$-1(x^2-18x+81)=0$

$-1(x-9)(x-9)=0$

$x=9$

Graphically



$f(x)$ is strictly less than 0 for all numbers except $x=9$, since $f(9)=0$. Therefore the solution is $(-\infty,9)\cup(9,\infty)$

6. $5x^2-2x \geq 16$

$\quad -16 \quad -16$

$5x^2-2x-16 \geq 0$

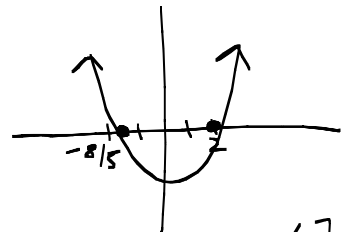
Find zeros of $f(x)=5x^2-2x-16$

$(5x+8)(x-2)=0$

$5x+8=0 \qquad x-2=0$

$x=-8/5 \qquad x=2$

Graphically



Solution: $(-\infty,-8/5]\cup[2,\infty)$

7. $0=x^4-25x^2+144$

Let $u=x^2$

Then $u^2=x^4$

$0=u^2-25u+144$ Factor

$0=(u-9)(u-16)$

$u=9, \quad u=16$ Since $u=x^2$

$x^2=9 \qquad x^2=16$

$x=\pm 3, \quad x=\pm 4$

8. $\frac{1}{x^2}+\frac{10}{x}+24=0 \qquad LCD = x^2$

$x^2\cdot\frac{1}{x^2}+x^2\frac{10}{x}+x^2\cdot 24=x^2\cdot 0$

$1+10x+24x^2=0$ reorder

$24x^2+10x+1=0$

$(6x+1)(4x+1)=0$

$6x+1=0 \qquad 4x+1=0$

$x=-1/6 \qquad x=-1/4$

9. $\dfrac{x}{x^2-16} + \dfrac{10}{x+4} = 1$

Factor

$\dfrac{x}{(x+4)(x-4)} + \dfrac{10}{x+4} = 1$

LCD $= (x+4)(x-4)$

$\cancel{(x+4)}\cancel{(x-4)} \cdot \dfrac{x}{\cancel{(x+4)}\cancel{(x-4)}} + \cancel{(x+4)}(x-4) \cdot \dfrac{10}{\cancel{x+4}} = (x+4)(x-4) \cdot 1$

$x + 10(x-4) = (x+4)(x-4)$

FOIL

$\underbrace{x + 10x}_{11x} - 40 = x^2 - 16$ ← Quadratic equation. Set one side equal to 0.

$0 = x^2 - 11x + 24$

$0 = (x-3)(x-8)$

$x = 3, \quad x = 8$

10. Let $u = t^3$

Then $u^2 = t^6$

$5u^2 + 15u = 9$ ← Quadratic equation in u.

$5u^2 + 15u - 9 = 0$ ← Not Factorable. Use the quadratic formula

$u = \dfrac{-15 \pm \sqrt{15^2 - 4(5)(-9)}}{2(5)}$

$u = \dfrac{-15 \pm \sqrt{405}}{10}$ → $u = 0.5125, \quad u = -3.5125$

$t^3 = 0.5125, \quad t^3 = -3.5125$ cube root

$t = 0.8002, \quad t = -1.5201$

11. Let $u = r^2$

Then $u^2 = r^4$

$u^2 - 29u + 100 = 0$

$(u-25)(u-4) = 0$

$u = 25, \quad u = 4$ since $u = r^2$

$r^2 = 25, \quad r^2 = 4$

$r = \pm 5, \quad r = \pm 2$

12. $\sqrt{-9-r} = 3$

$\left(\sqrt{-9-r}\right)^2 = (3)^2$

$-9 - r = 9$

$+9 \quad +9$

$-r = 18$ → $r = -18$

13. $t = \sqrt{12-t} + 12$ ↓ Isolate the radical

$t - 12 = \sqrt{12-t}$

$(t-12)^2 = (\sqrt{12-t})^2$

FOIL ↓ $t^2 - 12t - 12t + 144 = 12 - t$ ← Quadratic equation - Set one side equal to 0.

$t^2 - 23t + 132 = 0$

$(t-12)(t-11) = 0$

$t = 11,\ t = 12$

Check answers & you will find that $t=11$ doesn't work.
Solution : $t = 12$

14. $\dfrac{5}{3x+1} - \dfrac{24x}{3x-1} = -8$ ↓ LCD = $(3x+1)(3x-1)$

$\cancel{(3x+1)}(3x-1) \cdot \dfrac{5}{\cancel{3x+1}} - (3x+1)\cancel{(3x-1)} \cdot \dfrac{24x}{\cancel{3x-1}} = (3x+1)(3x-1) \cdot -8$

$5(3x-1) - 24x(3x+1) = -8\underbrace{(3x+1)(3x-1)}_{9x^2-1}$

$15x - 5 - 72x^2 - 24x = -72x^2 + 8$

$+72x^2 \qquad +72x^2$

$-9x - 5 = 8$

$-9x = 13$

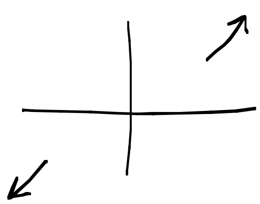
$x = -13/9$

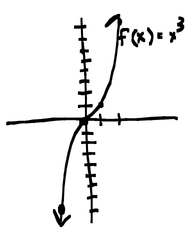
15. yes - it's smooth & continuous

16. The leading term is $\frac{7}{8}s^5$. The coefficient is positive; the exponent is odd.

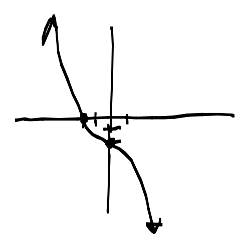
As $x \to \infty$, $f(x) \to \infty$

As $x \to -\infty$, $f(x) \to -\infty$



17. 

Take the graph of $f(x) = x^3$,
shift it left one unit, $f(x) = (x-1)^3$
reflect it across the x axis, $f(x) = -(x-1)^3$
and then shift it down one unit $f(x) = -(x-1)^3 - 1$



18. zeros $\pm 5, \pm 6$

Intervals	Test point	+/-
$(-\infty, -6)$	-7	$-$
$(-6, -5)$	-5.5	$+$
$(-5, 5)$	0	$-$
$(5, 6)$	5.5	$+$
$(6, \infty)$	7	$-$

19. To find the x-intercepts, find where $f(x) = 0$.

$$0 = -\frac{1}{9}(x^2-36)(x^2-25)$$

$$0 = -\frac{1}{9}(x+6)(x-6)(x+5)(x-5)$$

$$x = \pm 6,\ x = \pm 5$$

To find the y-intercept, find $f(0) = -\frac{1}{9}(0^2-36)(0^2-25) = -100$

20. $(0,3)$

21. Even

22.

zeros	multiplicity	crosses/touches
0	5	crosses
4	1	crosses
-5	1	crosses

23. symmetry about the y axis, since $f(-x) = f(x)$
even

24.

zeros	multiplicity
-1	odd
0	odd
3/2	odd

25.

x-intercepts	multiplicity
-7	3
4	3

26. $f(x) = (x-6)^2(x-3)^2$

27. $P(x) = x(x^2 - 5x - 24)$

$$P(x) = x(x-8)(x+3)$$

$$x = 0,\ x = 8,\ x = -3$$

28. zeros: $-2, 4$

Intervals	Test point	+/−
$(-\infty,-2)$	-3	$-$
$(-2,4)$	0	$-$
$(4,\infty)$	5	$-$

negative on $(-\infty,-2)$, $(-2,4)$, $(4,\infty)$

29. Enter into calculator & use $2^{nd} \rightarrow$ trace (CALC) $\rightarrow$ zero

$(-2.2618, 0)$, $(-0.3399, 0)$, $(2.6017, 0)$