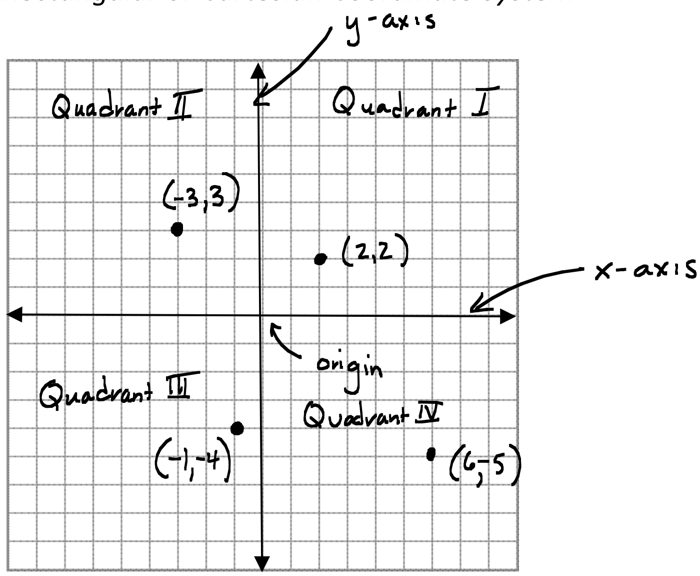
# Section 1.1
# Rectangular Coordinates and Graphing Utilities

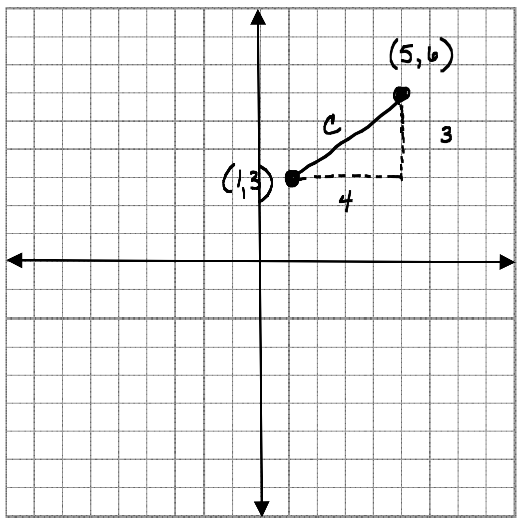
## Objectives
1. Use the distance formula
2. Use the midpoint formula

*Rectangular or Cartesian Coordinate System*



## Finding the Distance Between Two Points Using the Pythagorean Theorem

Example: Find the distance d between the points (1,3) and (5,6).




You could find the distance by using the Pythagorean Theorem.

$a^2 + b^2 = c^2$

$3^2 + 4^2 = c^2$

$9 + 16 = c^2$

$25 = c^2$

$5 = c$

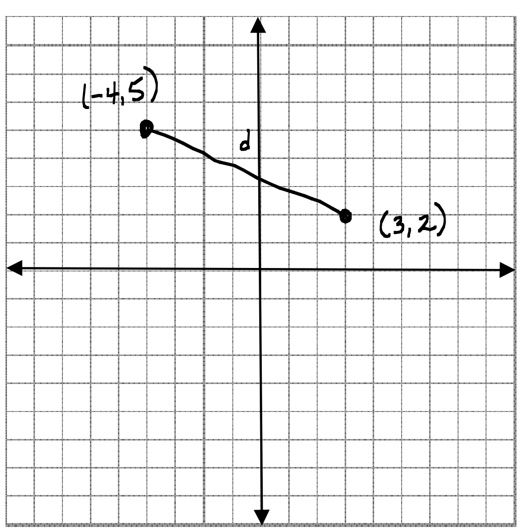
## The Distance Formula

The distance between two points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$, denoted by $d$, is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

This formula is based on the Pythagorean theorem.

Example: Find the length of the line segment from (-4, 5) to (3, 2).




Let Point 1 be $(\overset{x}{-4},\overset{y}{5})$

Then $x_1 = -4$ and $y_1 = 5$.

Let Point 2 be $(\overset{x}{3},\overset{y}{2})$

Then $x_2 = 3$ and $y_2 = 2$

$d = \sqrt{(3-(-4))^2 + (2-5)^2}$

$d = \sqrt{(3+4)^2 + (2-5)^2}$

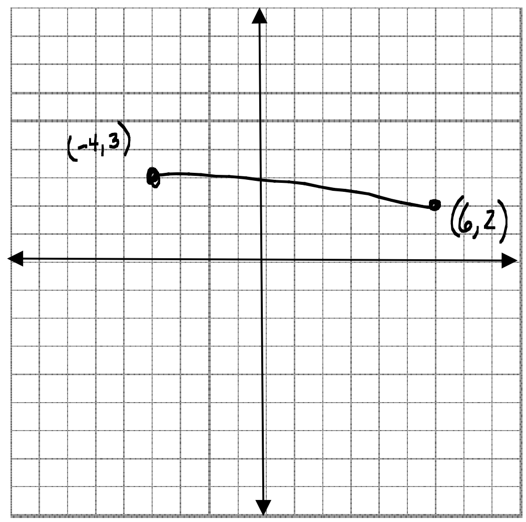
$d = \sqrt{7^2 + (-3)^2}$

$d = \sqrt{49 + 9}$

$d = \sqrt{58}$

Example: Find the length of the line segment from (-4, -3) to (6, 2).

$x_1, y_1$ $\quad$ $x_2, y_2$




$$d = \sqrt{(6-(-4))^2 + (2-(-3))^2}$$

$$d = \sqrt{(6+4)^2 + (2+3)^2}$$

$$d = \sqrt{10^2 + 5^2}$$

$$d = \sqrt{100 + 25}$$

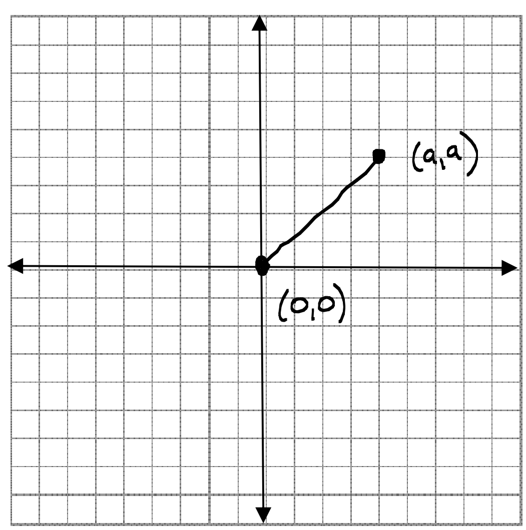
$$d = \sqrt{125} \leftarrow$$ Factor 125 as a perfect square times another number

$$d = \sqrt{25 \cdot 5}$$

$$d = \sqrt{25} \cdot \sqrt{5}$$

$$d = 5\sqrt{5}$$

Example: Find the length of the line segment from (a, a) to (0, 0).




Chose an arbitrary point where the x and y coordinates are the same for (a, a).

| Point 1 | Point 2 |
|---|---|
| (a, a) | (0, 0) |
| $x_1, y_1$ | $x_2, y_2$ |

$$d = \sqrt{(a-0)^2 + (a-0)^2}$$

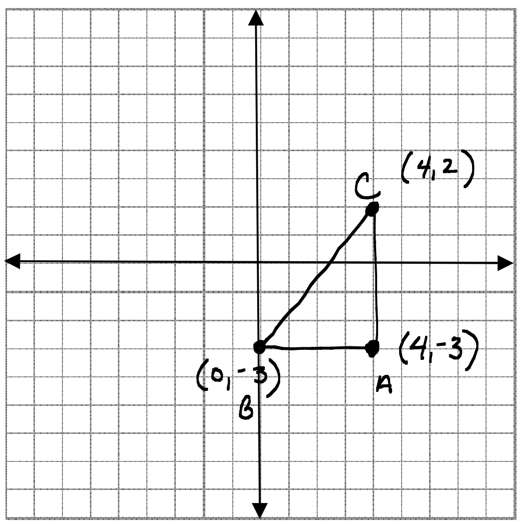
$$d = \sqrt{a^2 + a^2}$$

$$d = \sqrt{2a^2}$$

$$d = \sqrt{2}\, a$$

## Using Algebra to Solve Geometry Problems

Example: Plot each point and form the triangle ABC. Verify that the triangle is a right triangle and find its area. A = (4, -3); B = (0, -3); C = (4, 2).




First, find the lengths of the sides

$\overline{AC} = 5$

$\overline{AB} = 4$

Use distance formula to get

$$\overline{BC} = \sqrt{(4-0)^2 + (2-(-3))^2}$$
$$= \sqrt{4^2 + 5^2}$$
$$= \sqrt{16+25}$$
$$= \sqrt{41}$$

To verify that this is a right triangle, the sides should "work" in the Pythagorean Theorem

Does $5^2 + 4^2 = (\sqrt{41})^2$ ?

yes! It is a right triangle

## The Midpoint Formula

The midpoint $M = (x, y)$ of the line segment from $P_1 = (x_1, y_1)$ to $P_2 = (x_2, y_2)$ is

$$M = (x, y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

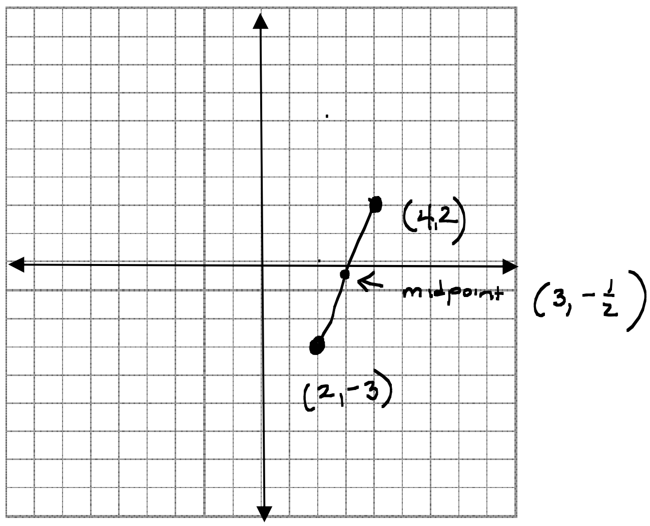
The average of the x coordinates

The average of the y coordinates

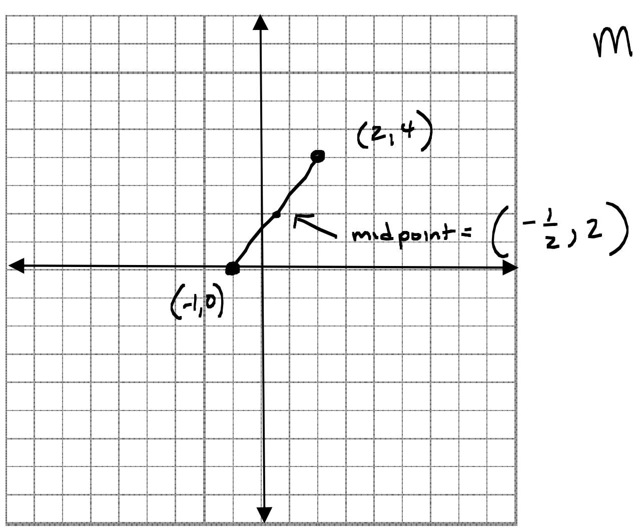
Example: Find the midpoint of the line segment joining the points $P_1 = (2, -3)$ and $P_2 = (4, 2)$.

$x_1\ y_1$ $\quad$ $x_2\ y_2$

$$M = (x, y) = \left(\frac{2+4}{2}, \frac{-3+2}{2}\right)$$
$$= \left(3, -\frac{1}{2}\right)$$




Example: Find the midpoint of the line segment joining the points $P_1 = (-1, 0)$ and $P_2 = (2, 4)$.




$$m = (x, y) = \left( \frac{-1+2}{2}, \frac{0+4}{2} \right)$$

$$= \left( \frac{1}{2}, 2 \right)$$