Section 2.3
Properties of Functions

Objectives
1. Determine Even and Odd Functions from a Graph
2. Identify Even and Odd Functions from the Equation
3. Use a Graph to Determine Where a Function is Increasing, Decreasing, or Constant
4. Use a Graph to Locate Local Maxima and Local Minima
5. Use a Graphing Calculator to Approximate Local Maxima and Local Minima and to Determine Where a Function is Increasing or Decreasing
6. Find the average rate of change of a function

A function f is <u>even</u> if, for every number x in its domain, the number $-x$ is also in the domain, and

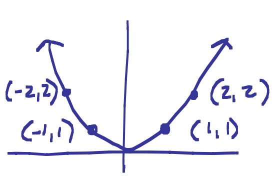
$$f(-x) = f(x)$$

A function f is <u>odd</u> if, for every number x in its domain, the number $-x$ is also in the domain, and

$$f(-x) = -f(x)$$

Theorem: A function is even if and only if its graph is symmetric with respect to the y-axis. A function is odd if and only if its graph is symmetric with respect to the origin.

Even Function

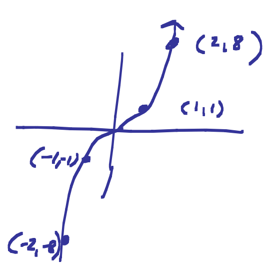


$f(1) = 1$
$f(-1) = 1$ ← $f(x) = f(-x)$

$f(2) = 2$
$f(-2) = 2$ ← $f(x) = f(-x)$

Odd Function

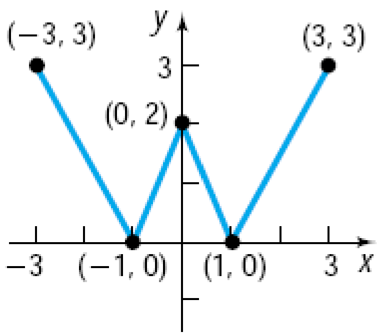
$f(-x) = -f(x)$



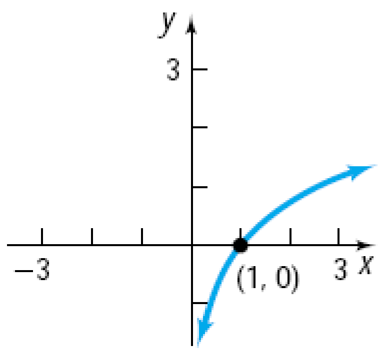
$f(-1) = -1$
$-f(1) = -1$ ← $f(-x) = -f(x)$

$f(-2) = -8$
$-f(2) = -8$ ← $f(-x) = -f(x)$

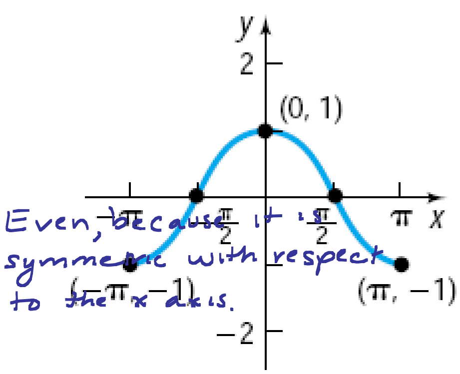
Example: Determine whether each graph is a graph of an even function, and odd function, or a function that is neither even or odd.



Even, because it is symmetric with respect to the y axis.



Neither



Even, because it is symmetric with respect to the y axis.

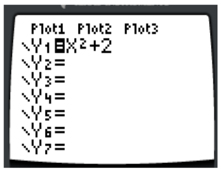
Identifying Even and Odd Functions from an Equation

Example: Are each of the following functions even, odd, or neither?

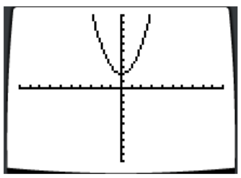
$f(x) = x^2 + 2$

Two ways

Calculator



Symm. with respect to y axis: even



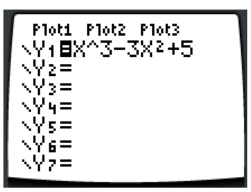
Algebraically

If the function is even, you should be able to plug in $-x$ and get $f(x)$.

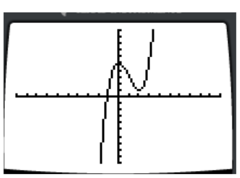
$f(-x) = (-x)^2 + 2 = \underbrace{x^2 + 2}_{f(x)}$

Even

$f(x) = x^3 - 3x^2 + 5$



Neither



Even? Does $f(-x) = f(x)$?

$f(-x) = (-x)^3 - 3(-x)^2 + 5$

$= -x^3 - 3x^2 - 5$ no

Odd? Does $f(-x) = -f(x)$?

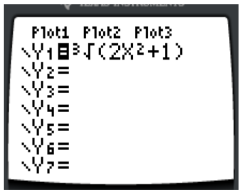
$f(-x) = -x^3 - 3x^2 - 5$ (from above)

$-f(x) = -x^3 + 3x^2 - 5$

no

Neither

$f(x) = \sqrt[3]{2x^2+1}$



Even? Does $f(-x) = f(x)$?

$f(-x) = \sqrt[3]{2(-x)^2+1} = \underbrace{\sqrt[3]{2x^2+1}}_{f(x)}$

Yes. Even

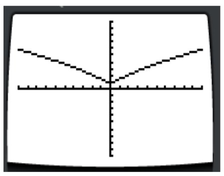
Odd? Does $f(-x) = -f(-x)$?

$f(-x) = \sqrt[3]{2x^2+1}$ (from above)

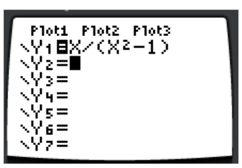
$-f(x) = -\sqrt[3]{2x^2+1}$

No. Odd

Symm. with respect to y axis: even →



$f(x) = \dfrac{x}{x^2-1}$



Even? Does $f(-x) = f(x)$?

$f(-x) = \dfrac{-x}{(-x)^2-1} = \dfrac{-x}{x^2-1}$

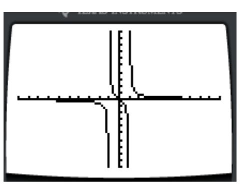
No. Not even.

Odd? Does $f(-x) = -f(x)$?

$f(-x) = \dfrac{-x}{x^2-1}$ (from above)

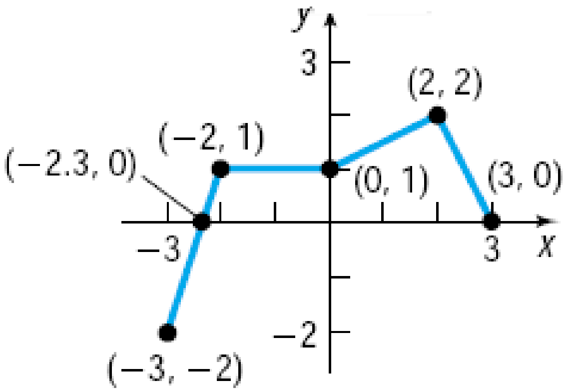
$-f(x) = -\dfrac{x}{x^2-1}$ Yes: Odd.

Symm. with respect to the origin: Odd →



Using a Graph to Determine Where a Function is Increasing, Decreasing or Constant

Example:



Where is the function increasing?

For what values of x is the function increasing?

$-3 < x < -2$ $(-3, -2)$

$0 < x < 2$ $(0, 2)$

Where is the function decreasing?

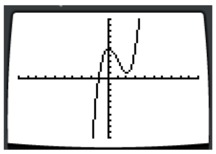
$2 < x < 3$ $(2, 3)$

Where is the function constant?

$-2 < x < 0$ $(-2, 0)$

Using a Graphing Calculator to Approximate Local Maxima and Local Minima and to Determine Where a Function is Increasing or Decreasing

Example: Graph the function $f(x) = x^3 - 3x^2 + 5$ over the interval (-1, 3) and approximate any local maxima and minima. Determine where the function is increasing and where it is decreasing.

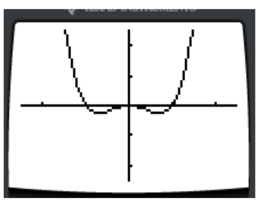


Use MAXIMUM and MINIMUM.

Local Minimum: 1, when $x = 2$

Local Maximum: 5, when $X = 0$

Example: Graph the function $f(x) = x^4 - x^2$ over the interval (-2, 2) and approximate any local maxima and minima. Determine where the function is increasing and where it is decreasing.



Local Maximum: 0, when $x = 0$

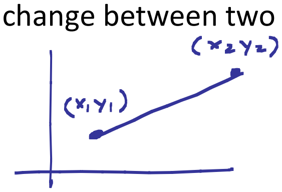
Local Minima: $-.25$ when $x = -.71$

$-.25$ when $x = .71$

Find the Average Rate of Change of a Function

Recall that the average rate of change between two points is the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

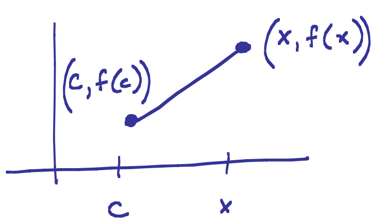


If c is in the domain of a function $y = f(x)$, the average rate of change of f from c to x is defined as

$$\text{Average rate of change} = \frac{\Delta y}{\Delta x} = \frac{f(x) - f(c)}{x - c}$$

This is called the <u>difference quotient</u>.

$$m = \frac{f(x) - f(c)}{x - c}$$



Example: Find the average rate of change of $f(x) = -x^3 + 1$

a. From 0 to 2

x	$f(x)$	
0	1	$\leftarrow -(0)^3+1$
2	-7	$\leftarrow -(2)^3+1$

$$m = \frac{1-(-7)}{2-0} = \frac{1+7}{2} = \frac{8}{2} = 4$$

b. From 1 to 3

x	$f(x)$	
1	0	$\leftarrow -1^3+1$
3	-26	$\leftarrow -3^3+1$

$$m = \frac{0-(-26)}{1-3} = \frac{26}{-2} = 13$$

c. From -1 to 1

x	$f(x)$	
-1	2	$\leftarrow -(-1)^3+1$
1	0	$\leftarrow -1^3+1$

$$m = \frac{2-0}{-1-1} = \frac{2}{-2} = -1$$