

## Section 2.5

### Library of Functions; Piecewise-defined Functions

#### Objectives

1. Graph the Functions Listed in the Library of Functions
2. Graph Piecewise-defined Functions

Library of Functions—a set of the most frequently used basic functions.

#### Steps to Graphing Functions

1. Find the intercepts.
2. Determine if the function is even, odd, or neither.
3. Graph the function by plotting points.
4. Determine where the function is increasing and decreasing.
5. Identify any local maxima or minima.

**Linear Function:**  $f(x) = mx + b$

x-intercept: Set  $y$  (or  $f(x)$ ) to 0  
and solve for  $x$ .

$$0 = mx + b$$

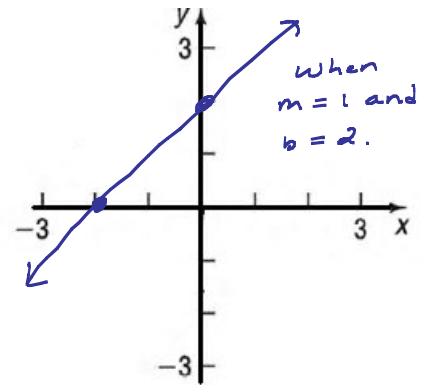
$$\frac{-b}{m} = \frac{mx}{m}$$

$$\frac{-b}{m} = x$$

y-intercept: Set  $x = 0$

$$f(0) = m(0) + b = b$$

Neither even nor odd.

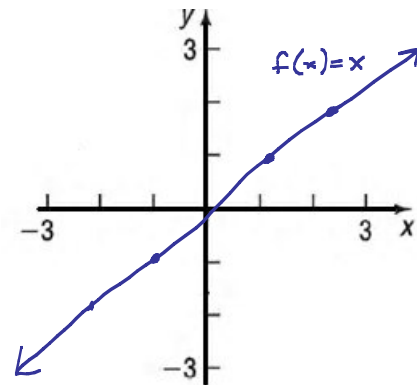


**Identity Function:**  $f(x) = x$

Intercepts:  $(0, 0)$

Odd function, since the graph  
is symmetric with respect to  
the origin.

Increasing on  $(-\infty, \infty)$



**Square Function:**  $f(x) = x^2$

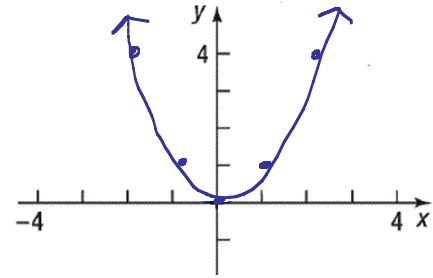
Intercept:  $(0, 0)$

Even: Symmetric with respect to the y-axis

Decreasing:  $(-\infty, 0)$

Increasing:  $(0, \infty)$

Local minimum at 0 when  $x=0$

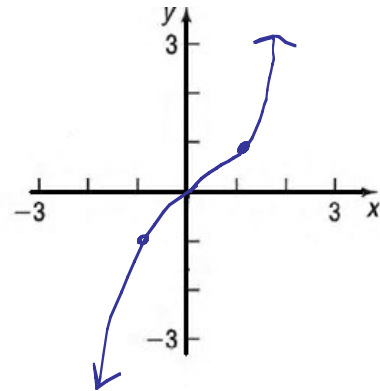


**Cube Function:**  $f(x) = x^3$

Intercept:  $(0, 0)$

Odd: Symmetric with respect to the origin.

Increasing on  $(-\infty, \infty)$

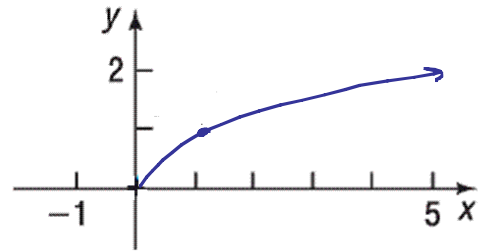


**Square Root Function:**  $f(x) = \sqrt{x}$

Intercept:  $(0, 0)$

Neither even nor odd

Increasing:  $(0, \infty)$

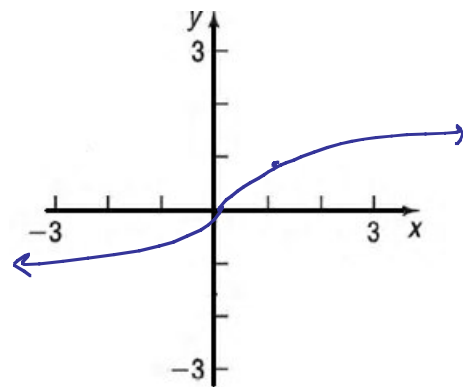


**Cube Root Function:**  $f(x) = \sqrt[3]{x}$

Intercept:  $(0, 0)$

Odd since the graph is symmetric with respect to the origin.

Increasing on  $(-\infty, \infty)$



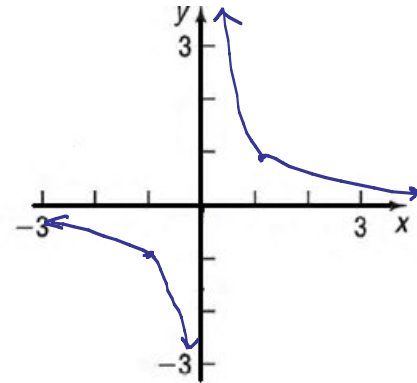
Reciprocal Function:  $f(x) = \frac{1}{x}$

No intercepts.

Odd since the graph is symmetric with respect to the origin.

Decreasing on  $(-\infty, 0)$

Increasing on  $(0, \infty)$



Absolute Value Function:  $f(x) = |x|$

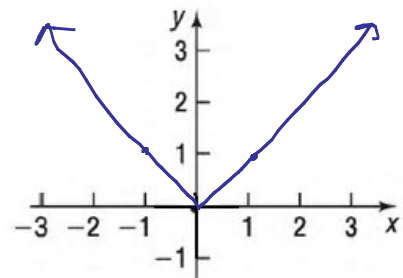
Intercept:  $(0, 0)$

Even since the graph is symmetric with respect to the y axis.

Decreasing  $(-\infty, 0)$

Increasing  $(0, \infty)$

Local minimum at 0 when  $x=0$

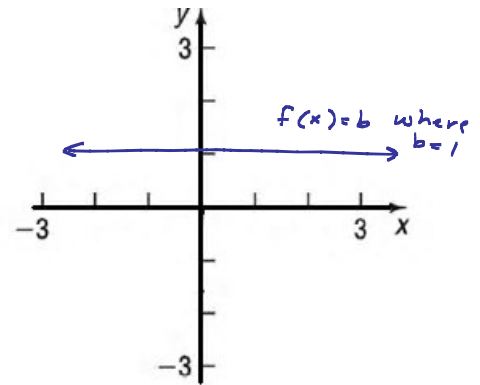


**Constant Function:  $f(x) = b$**

y intercept =  $b$

Even, since the graph is symmetric with respect to the y axis.

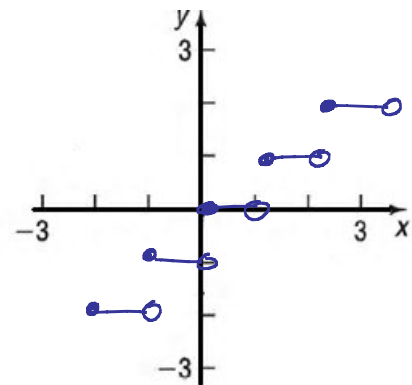
Constant on  $(-\infty, \infty)$



**Greatest Integer Function**

$f(x) = \text{int}(x) =$  greatest integer less than or equal to  $x$   
(round down to nearest integer)

x	f(x)
0	0
-1	0
-2	0
-3	0
-4	0
-5	0
-6	0
-7	0
-8	0
-9	0
-10	0
-11	0
-12	0
-13	0
-14	0
-15	0
-16	0
-17	0
-18	0
-19	0
-20	0
-21	0
-22	0
-23	0
-24	0
-25	0
-26	0
-27	0
-28	0
-29	0
-30	0
-31	0
-32	0
-33	0
-34	0
-35	0
-36	0
-37	0
-38	0
-39	0
-40	0
-41	0
-42	0
-43	0
-44	0
-45	0
-46	0
-47	0
-48	0
-49	0
-50	0
-51	0
-52	0
-53	0
-54	0
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-56	0
-57	0
-58	0
-59	0
-60	0
-61	0
-62	0
-63	0
-64	0
-65	0
-66	0
-67	0
-68	0
-69	0
-70	0
-71	0
-72	0
-73	0
-74	0
-75	0
-76	0
-77	0
-78	0
-79	0
-80	0
-81	0
-82	0
-83	0
-84	0
-85	0
-86	0
-87	0
-88	0
-89	0
-90	0
-91	0
-92	0
-93	0
-94	0
-95	0
-96	0
-97	0
-98	0
-99	0
-100	0



## Piecewise-defined Functions

Example: The function  $f$  is defined as

$$f(x) = \begin{cases} x + 3 & \text{if } x < -2 \\ -2x - 3 & \text{if } x \geq -2 \end{cases}$$

← If  $x$  is less than  $-2$ , use this rule  
← If  $x$  is greater than or equal to  $-2$ , use this rule.

Find the domain and graph.

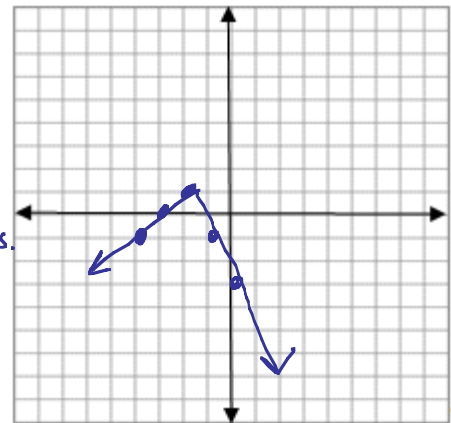
$x$	$f(x)$	
-4	-1	← 1 <sup>st</sup> rule
-3	0	← 1 <sup>st</sup> rule
-2	1	← 2 <sup>nd</sup> rule
-1	-1	← 2 <sup>nd</sup> rule
0	-3	← 2 <sup>nd</sup> rule

Domain: All real numbers

Range:  $(-\infty, 1]$

↑  
y-coordinates  
on the graph

Notes the arrows.  
The function  
is defined for  
all values of  
 $x$



Example: The function  $f$  is defined as

$$f(x) = \begin{cases} x + 3 & \text{if } -3 \leq x < 0 \\ 3 & \text{if } x = 0 \\ \sqrt{x} & \text{if } x > 0 \end{cases}$$

$\leftarrow$  1<sup>st</sup> rule  
 $\leftarrow$  2<sup>nd</sup> rule  
 $\leftarrow$  3<sup>rd</sup> rule

Find the domain and graph.

$x$	$f(x)$	
-3	0	$\leftarrow$ 1 <sup>st</sup>
-2	1	$\leftarrow$ 1 <sup>st</sup>
-1	2	$\leftarrow$ 1 <sup>st</sup>
0	3	$\leftarrow$ 2 <sup>nd</sup>
1	1	$\leftarrow$ 3 <sup>rd</sup>
2	$\sqrt{2}$	$\leftarrow$ 3 <sup>rd</sup>
3	$\sqrt{3}$	$\leftarrow$ 3 <sup>rd</sup>

Domain:  $[-3, \infty)$

Range:  $[0, \infty)$

