Section 2.6
Graphing Techniques: Transformations

Objectives
1. Graph Functions Using Vertical and Horizontal Shifts.
2. Graph Functions Using Vertical Compressions and Stretches.
3. Graph Functions Using Reflections about the x-Axis and y-Axis.

Recall: The library of functions:
$f(x) = x^2$
$f(x) = x^3$
$f(x) = \sqrt{x}$
$f(x) = \sqrt[3]{x}$
$f(x) = |x|$
$f(x) = \frac{1}{x}$

<u>Transformations</u>—a collection of techniques for graphing functions that look "almost" like those in the library of functions.

Examples:

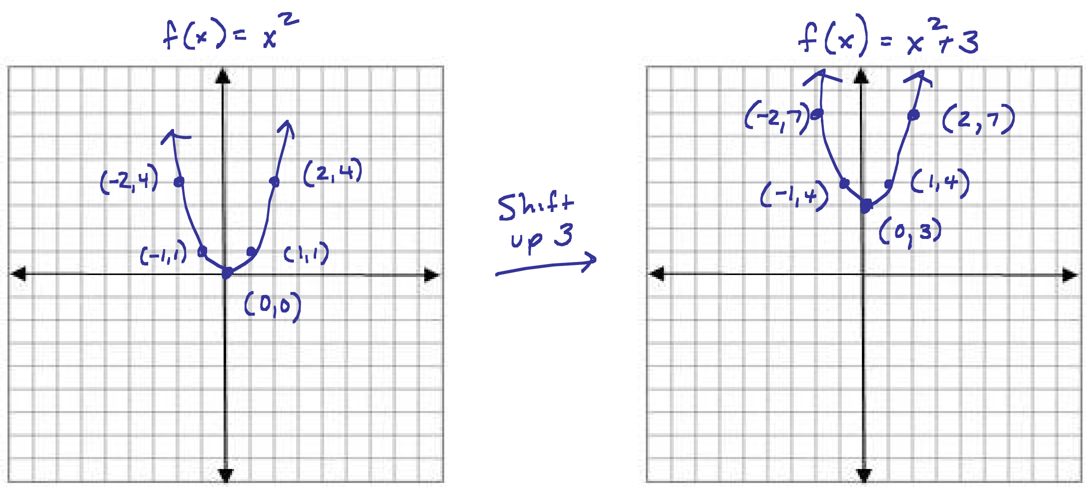
$f(x) = (x+1)^2$ $\qquad$ $f(x) = \frac{2}{x} + 4$

$f(x) = \frac{1}{3}\sqrt{x}$ $\qquad$ $f(x) = 5\sqrt[3]{x+1}$

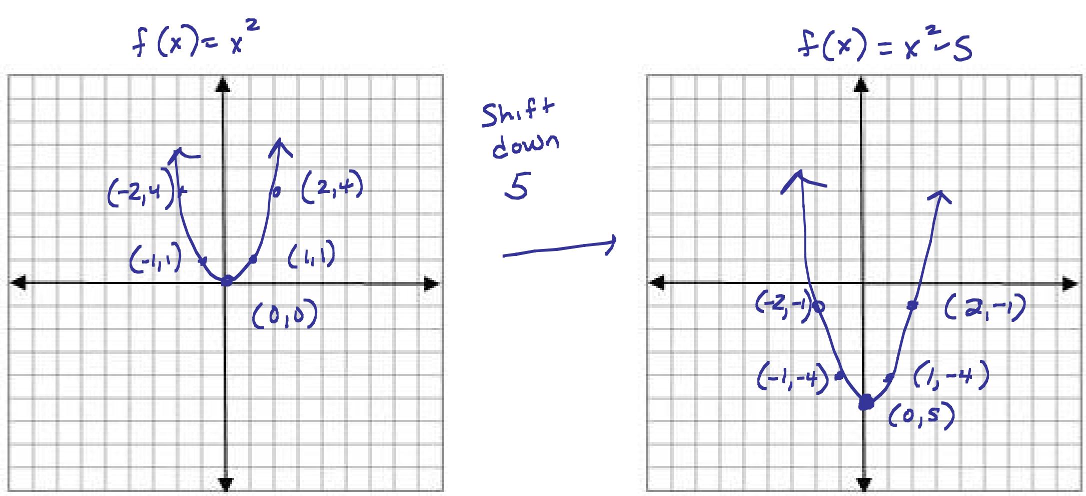
Vertical Shifts

If a real number k is added to the right side of a function y = f(x), the graph of the new function y= f(x) + k is the graph of f shifted vertically up k units (if k > 0) or down |k| units (if k < 0).

Example: Use the graph of $f(x) = x^2$ to obtain the graph of $f(x) = x^2 + 3$.



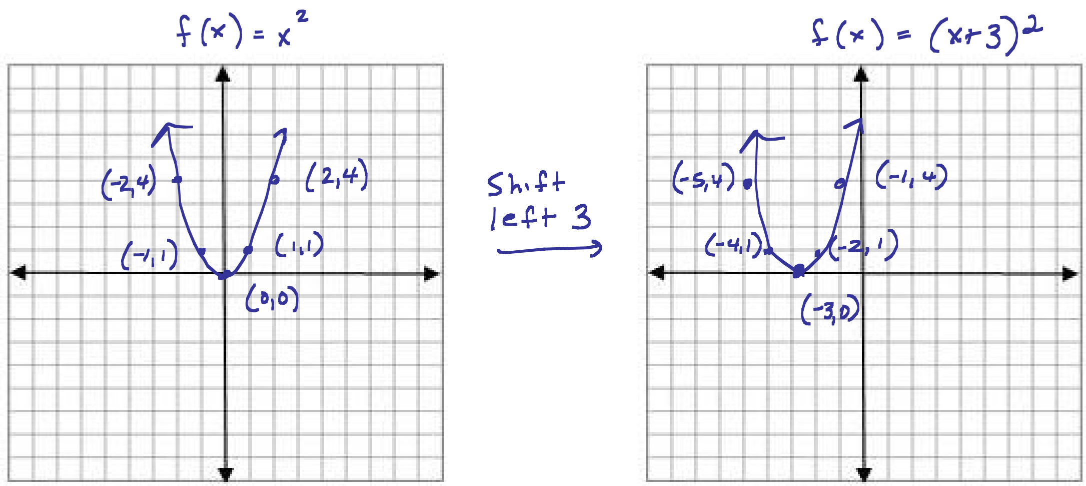
Example: Use the graph of $f(x) = x^2$ to obtain the graph of $f(x) = x^2 - 5$.



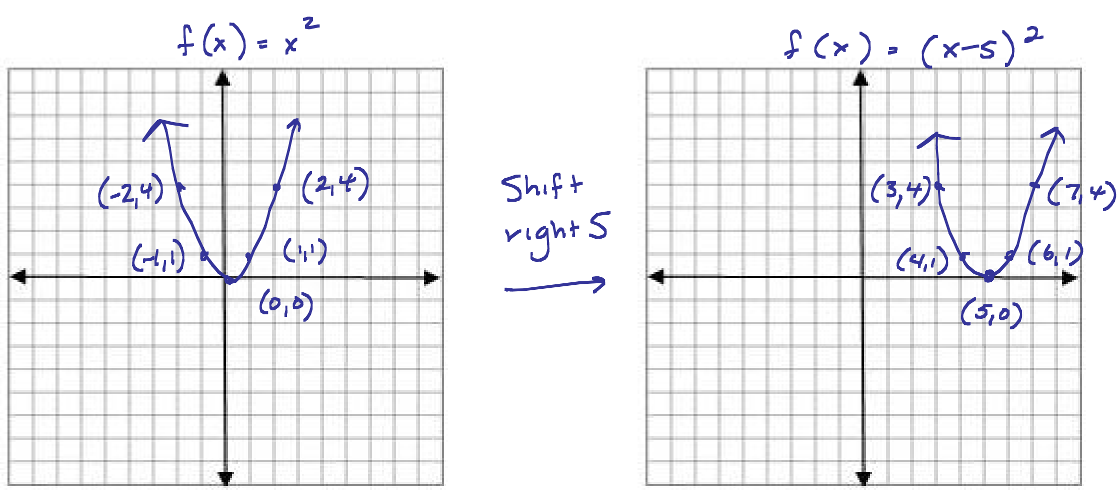
Horizontal Shifts

If the argument x of a function f is replaced by x - h, h > 0, the graph of the new function y = f(x-h) is the graph of f shifted horizontally right h units. If the argument x of a function f is replaced by x + h, h > 0, the graph of the new function y = f(x + h) is the graph of f shifted horizontally left h units.

Example: Use the graph of $f(x) = x^2$ to obtain the graph of $f(x) = (x + 3)^2$



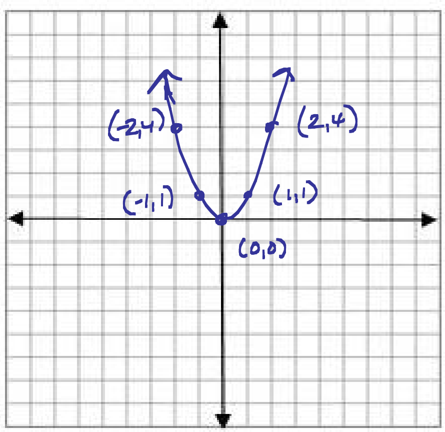
Example: Use the graph of $f(x) = x^2$ to obtain the graph of $f(x) = (x - 5)^2$



Combining Vertical and Horizontal Shifts

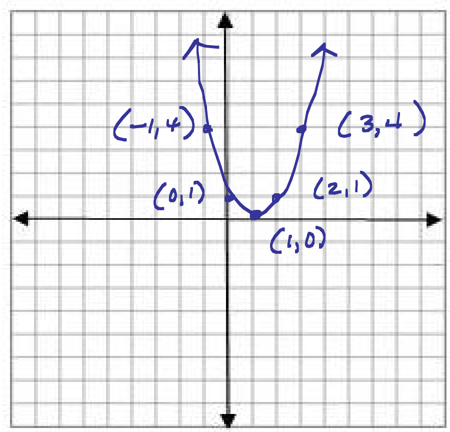
Example: Graph the function $f(x) = (x-1)^2 + 3$ using vertical and horizontal shifts.

$f(x) = x^2$



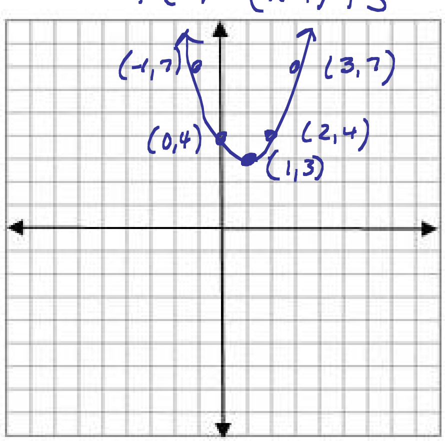
Shift right 1

$f(x) = (x-1)^2$



shift up 3

$f(x) = (x-1)^2 + 3$

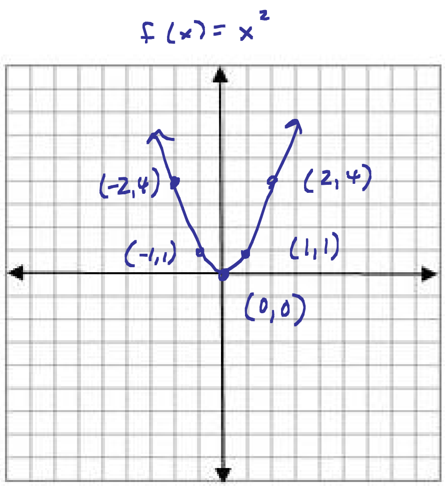


Graphing Functions Using Compressions and Stretches

Vertical Compressions and Stretches

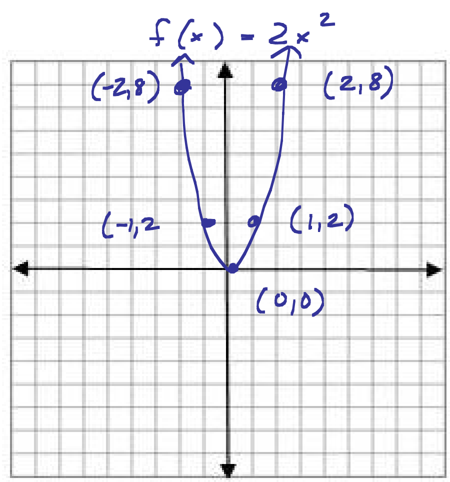
When the right side of a function $y = f(x)$ is multiplied by a positive number a, the graph of the new function $y = af(x)$ is obtained by multiplying each y-coordinate on the graph of $y = f(x)$ by a. The new graph is **vertically compressed** (if $0 < a < 1$) or **vertically stretched** (if $a > 1$).

Example: Use the graph of $f(x) = x^2$ to obtain the graph of $f(x) = 2x^2$.



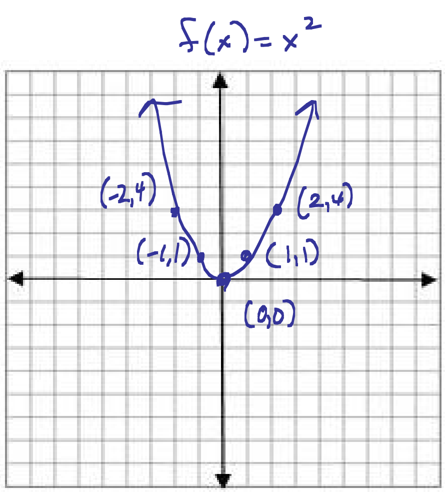
Vertical Stretch by a factor of 2.

Multiply each y coordinate by 2.



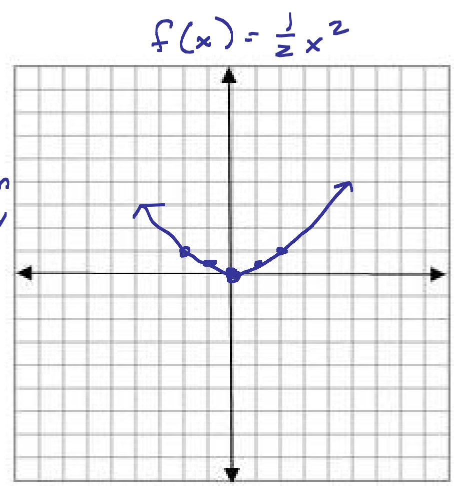
This graph increases twice as fast as the first one.

Example: Use the graph of $f(x) = x^2$ to obtain the graph of $f(x) = \frac{1}{2}x^2$.



Vertical Compression by a factor of ½

Multiply each y coordinate by ½



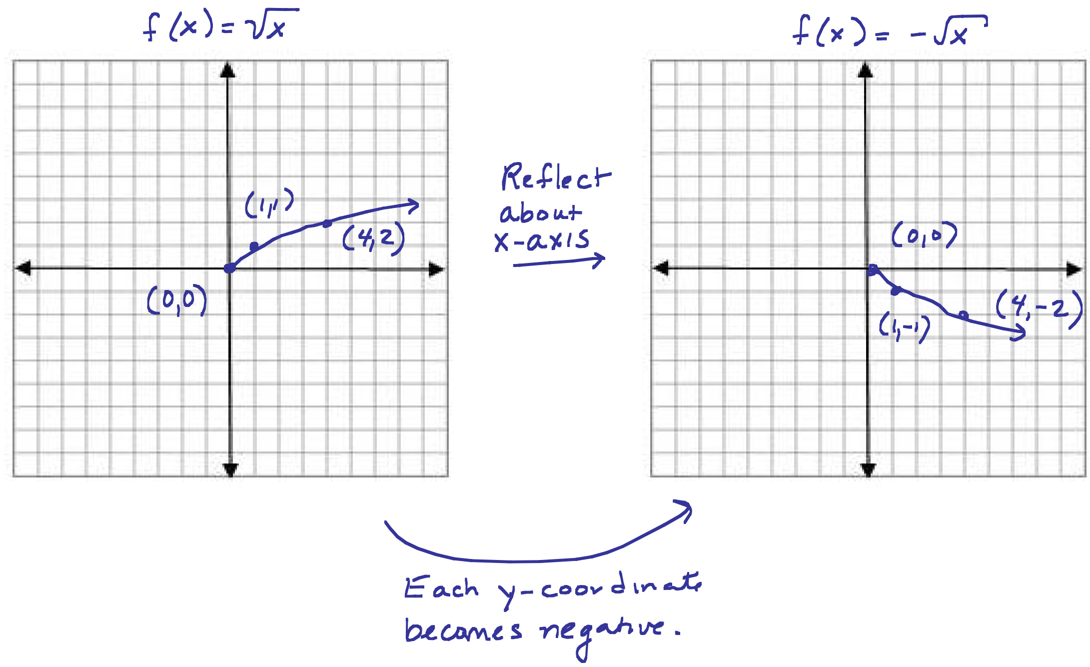
This graph increases ½ as fast as the original.

Graph Functions Using Reflections about the x-Axis or y-Axis

Reflections about the x-axis

When the right side of the function $y = f(x)$ is multiplied by -1, the graph of the new function $y = -f(x)$ is the reflection about the x-axis of the graph of the function $y = f(x)$.

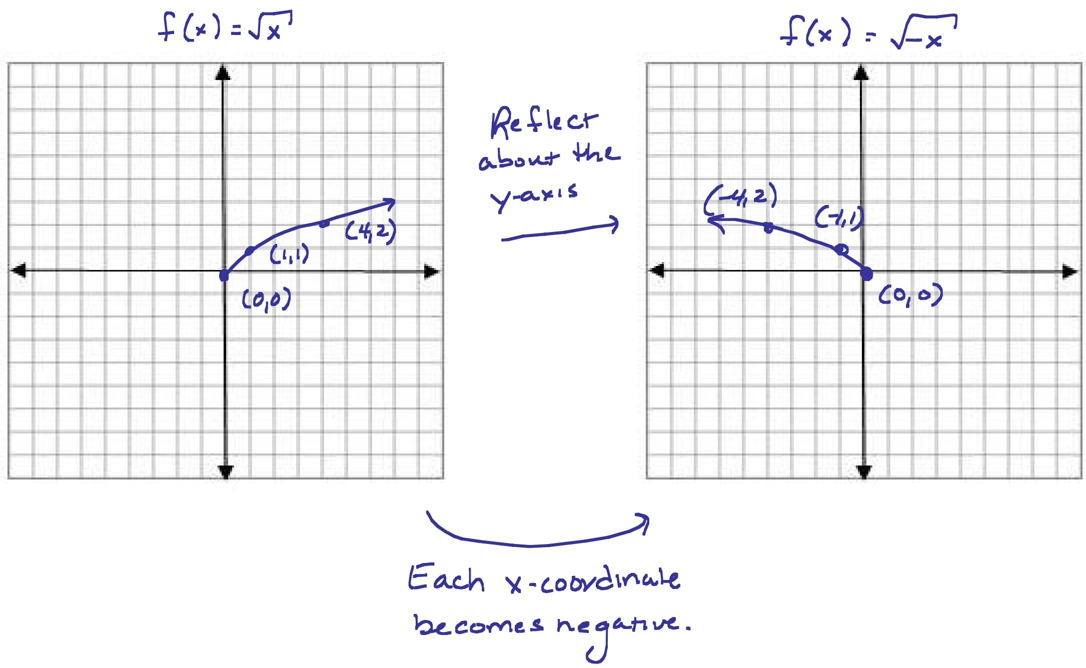
Example: Use the graph of $f(x) = \sqrt{x}$ to obtain the graph of $f(x) = -\sqrt{x}$.



Reflections about the y-axis

When the graph of the function $y = f(x)$ is known, the graph of the new function $y = f(-x)$ is the reflection about the y-axis of the graph of the function $y = f(x)$.

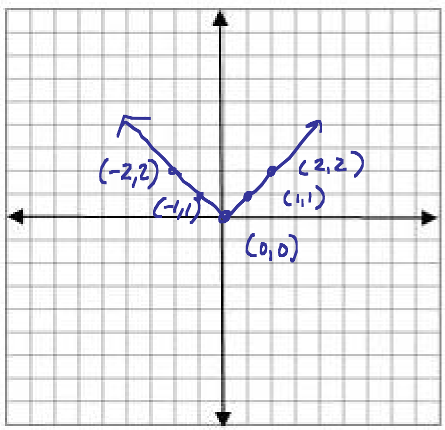
Example: Use the graph of $f(x) = \sqrt{x}$ to obtain the graph of $f(x) = \sqrt{-x}$.



Combining Graphing Procedures

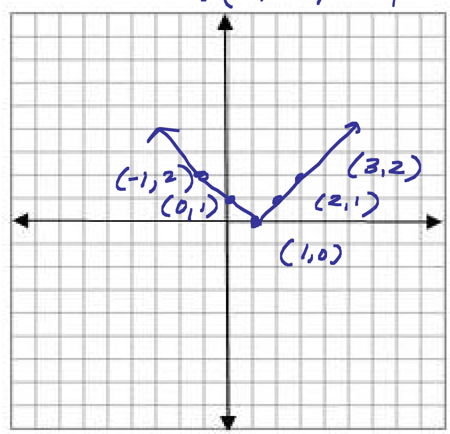
Example: Graph $f(x) = 2|x - 1|$.

$f(x) = |x|$



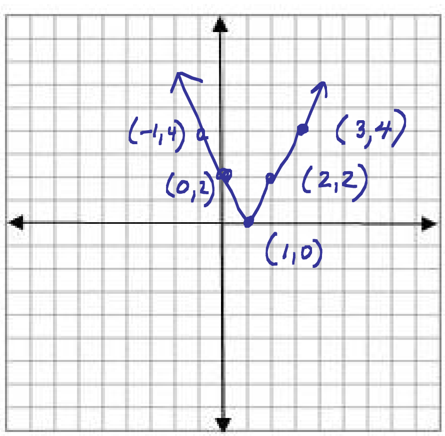
Shift right 1

$f(x) = |x-1|$

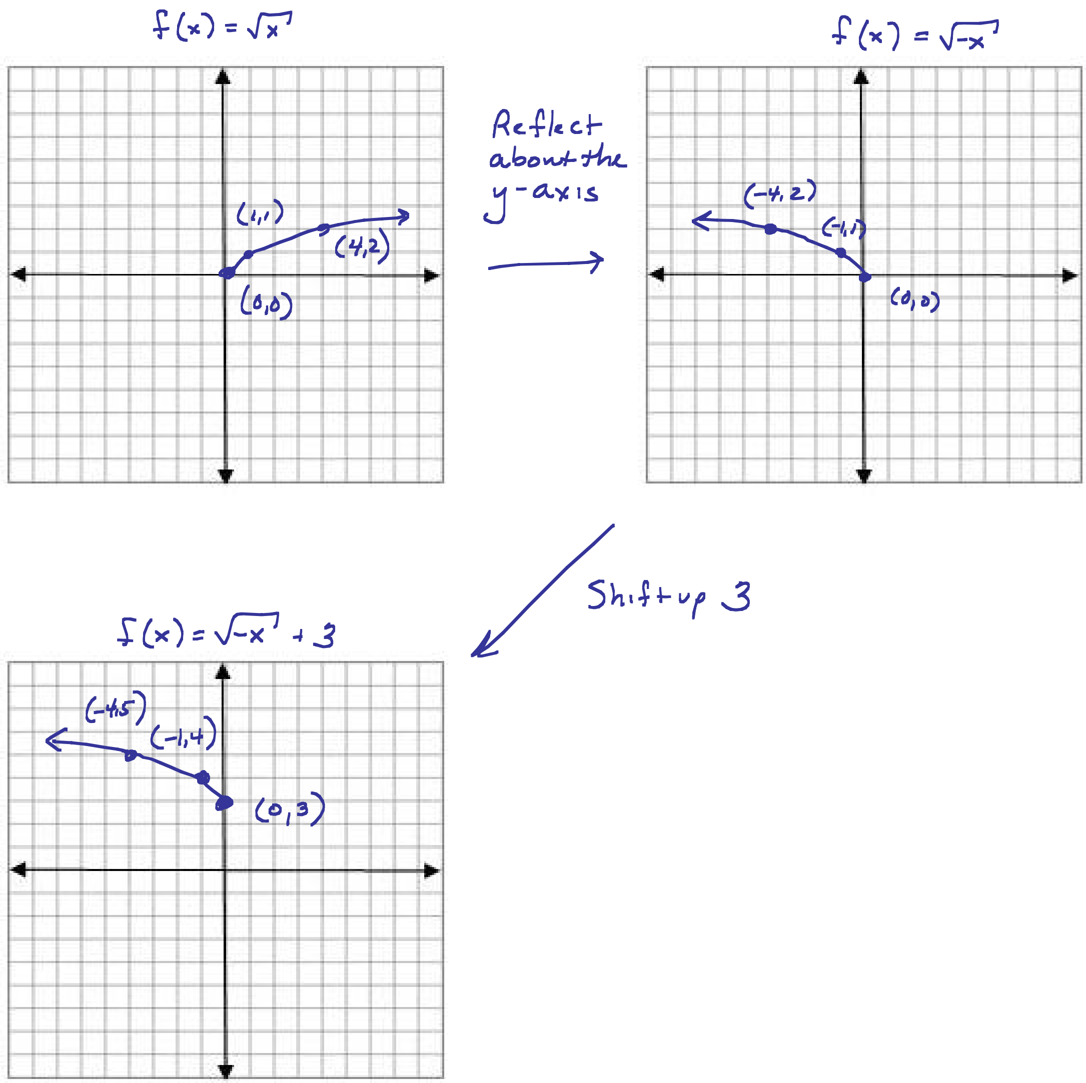


Vertical stretch by a factor of 2. Multiply each y-coordinate by 2

$f(x) = 2|x+1|$

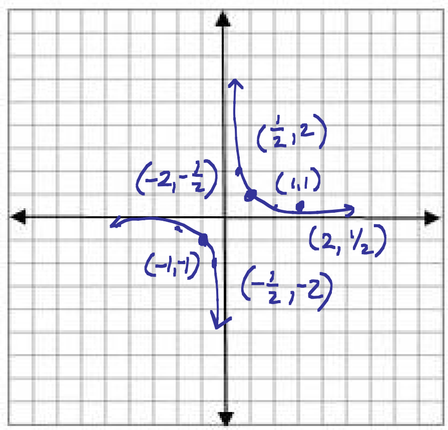


Example: Graph $f(x) = \sqrt{-x} + 3$.



Example: Graph $f(x) = \frac{1}{-x} + 2$

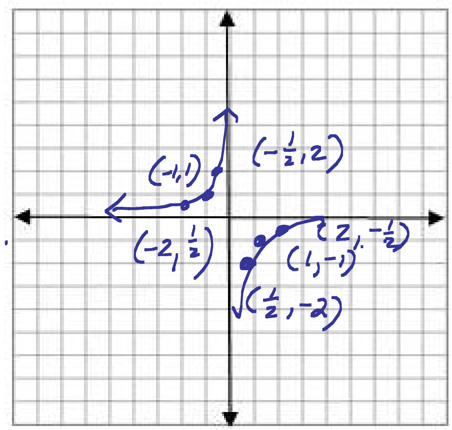
$f(x) = \frac{1}{x}$



Reflect about the y-axis

All y-coord. become the opposite sign.

$f(x) = -\frac{1}{x}$



Shift up 2

$f(x) = -\frac{1}{x} + 2$