Section 4.1
Composite Functions

Objectives

1. Form a Composite Function
2. Find the Domain of a Composite Function

Recall: To evaluate a function at a given value, replace x with that value.

For the function $f(x) = x + 1$,

$f(3) =$ $3 + 1 = 4$

$f(-4) =$ $-4 + 1 = -3$

In general, this function means every time you put in a number, this function returns one more than a number.

For the function $g(x) = x^2$,

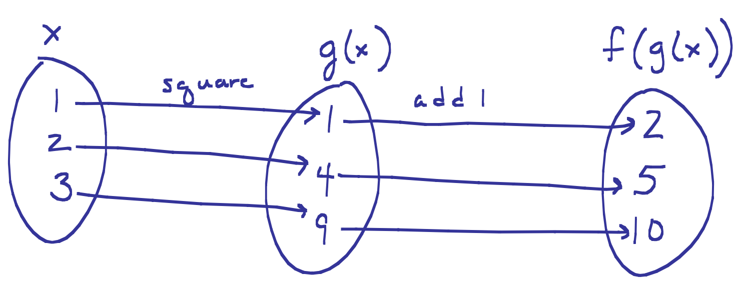
$g(3) =$ $3^2 = 9$

$g(-2) =$ $(-2)^2 = 4$

In general, this function means every time you put in a number, this function returns the square of that number.

Composition of functions takes two functions and makes a new one out of them.

Example: A function is created where you take a number, square it, and then add 1 to the result.



Composite Function

Given two functions f and g, the composite function, denoted by $f°g$, is defined by

$$(f°g)(x) = f(g(x))$$

To evaluate $(f°g)(4)$,

1. Rewrite $(f°g)(4)$ as $f\big(g(x)\big)$.
1. Find $g(x)$.
2. Take the value of $g(x)$and plug it into $f(x)$

Example: If $f(x) = x + 1$, and $g(x) = x^2$, then

Find $(f°g)(4)$.

$(f \circ g)(4) = f(g(4)) = f(16) = 16+1 = 17$

$g(4) = 4^2 = 16$

Find $(f°g)(2)$. $= f(4) = 4+1 = 5$

$(f \circ g)(2) = f(g(2)) =$

$g(2) = 2^2 = 4$

Example: Suppose $f(x) = 2x^2$ and $g(x) = 1 - 3x^2$.

Find $(f^\circ g)(3)$.

$$(f \circ g)(3) = f(g(3)) = f(-26) = 2(-26)^2 = 1352$$

$$g(3) = 1 - 3 \cdot 3^2 = 1 - 27 = -26$$

Find $(f^\circ g)(2)$.

$$(f \circ g)(2) = f(g(2)) = f(-11) = 2(-11)^2 = 2 \cdot 121 = 242$$

$$g(2) = 1 - 3(2)^2 = 1 - 12 = -11$$

Find $(f^\circ g)(-1)$.

$$(f \circ g)(x) = f(g(-1)) = f(-2) = 2(-2)^2 = 8$$

$$g(-1) = 1 - 3(-1)^2 = -2$$

Find $(f^\circ g)(-2)$.

$$(f \circ g)(-2) = f(g(-2)) = f(-11) = 2(-11)^2 = 242$$

$$g(-2) = 1 - 3(-2)^2 = -11$$

Find $(f^\circ g)(x)$.

substitute $1-3x^2$ for x in $f(x)$

$$(f \circ g)(x) = f(g(x)) = f(\underbrace{1 - 3x^2}_{g(x)}) = 2(1 - 3x^2)^2$$

$$= 2(1 - 3x^2)(1 - 3x^2)$$

$$= 2(1 - 3x^2 - 3x^2 + 9x^4)$$

$$= 2 - 12x^2 + 18x^4$$

Example: Suppose $f(x) = \sqrt{x+1}$ and $g(x) = 2x$.

Find $(f{\circ}g)(0)$.

$(f \circ g)(0) = f(g(0)) = f(0) = \sqrt{0+1} = \sqrt{1} = 1$

$g(0) = 2 \cdot 0 = 0$

Find $(f{\circ}g)(4)$.

$(f \circ g)(4) = f(g(4)) = f(8) = \sqrt{8+1} = \sqrt{9} = 3$

$g(4) = 2 \cdot 4 = 8$

Find $(f{\circ}g)(-12)$.

$(f \circ g)(-12) = f(g(-12)) = f(-24) = \sqrt{-24+1} = \cancel{\sqrt{-23}}$

Not a real number

-12 is not in the domain of $(f \circ g)(x)$

Find $(f{\circ}g)(x)$.

$g(x)$

Substitute $2x$ for x in $f(x)$

$(f \circ g)(x) = f(g(x)) = f(2x) = \sqrt{2x+1}$

Example: Suppose $f(x) = |x-2|$ and $g(x) = \frac{3}{x^2-1}$.

Find $(f\circ g)(3)$.

$$(f\circ g)(3) = f(g(3)) = f\left(\tfrac{3}{8}\right) = \left|\tfrac{3}{8} - 2\right|$$

$$= \left|\tfrac{3}{8} - \tfrac{16}{8}\right|$$

$$= \left|-\tfrac{13}{8}\right|$$

$$= \tfrac{13}{8}$$

Find $(f\circ g)(2)$.

$$(f\circ g)(2) = f(g(2)) = f(1) = |1-2| = |-1| = 1$$

$$g(2) = \frac{3}{2^2-1} = 1$$

Find $(f\circ g)(-1)$.

$$(f\circ g)(-1) = f(g(-1)) =$$

$g(-1) = \frac{3}{(-1)^2-1} = \frac{3}{1-1} = \frac{3}{0}$ not allowed!

-1 is not in the domain of $(f\circ g)(-1)$

Find $(f\circ g)(1)$.

$$(f\circ g)(1) = f(g(1)) =$$

$g(1) = \frac{3}{1^2-1} = \frac{3}{1-1} = \frac{3}{0}$ not allowed!

1 is not in the domain of $(f\circ g)(1)$

Example: Suppose that $f(x) = x - 2$ and $g(x) = \sqrt{1-x}$.

Find $(f^\circ g)(x)$.

$$(f \circ g)(x) = f(g(x)) = f\Big(\overbrace{\sqrt{1-x}}^{g(x)}\Big) = \sqrt{x-2} - 2$$

Find $(g^\circ f)(x)$.

$$(g \circ f)(x) = g(f(x)) = g\big(\overbrace{x-2}^{f(x)}\big) = \sqrt{1-(x-2)}$$

$$= \sqrt{1-x+2}$$

$$= \sqrt{3-x}$$

Find $(f^\circ f)(x)$.

$$(f \circ f)(x) = f(f(x)) = f(x-2) = (x-2)-2 = x-4$$

Find $(g^\circ g)(x)$.

$$(g \circ g)(x) = g(g(x)) = g\big(\sqrt{1-x}\big) = \sqrt{1-\sqrt{1-x}}$$

Example: Suppose that $f(x) = \dfrac{1}{x-1}$ and $g(x) = \dfrac{-4}{x}$

Find $(f^{\circ}g)(x)$.

mult. numerator + denominator by x, the LCD

$$(f\circ g)(x) = f(g(x)) = f\left(-\frac{4}{x}\right) = \frac{1}{-\frac{4}{x}-1} = \frac{x}{-4-x}$$

Find $(g^{\circ}f)(x)$.

$$(g\circ f)(x) = g(f(x)) = g\left(\frac{1}{x-1}\right) = \frac{-4}{\frac{1}{x-1}} = (-4)(x-1) = -4x+4$$

$$-4 \div \frac{1}{x-1} = -4 \cdot \frac{x-1}{1}$$

Find $(f^{\circ}f)(x)$.

Multiply numerator and denominator by x−1, the LCD

$$(f\circ f)(x) = f(f(x)) = f\left(\frac{1}{x-1}\right) = \frac{1}{\frac{1}{x-1}-1} = \frac{x-1}{1-(x-1)} = \frac{x-1}{-x}$$

Find $(g^{\circ}g)(x)$.

$$(g^{\circ}g)(x) = g(g(x)) = g\left(-\frac{4}{x}\right) = \frac{-4}{-\frac{4}{x}} = \frac{-4x}{-4} = x$$

$$-4 \div -\frac{4}{x} = -4 \cdot \frac{x}{-4}$$