Section A-1
Algebra Review

Objectives

1. Evaluate Algebraic Expressions
2. Determine the Domain of a Variable
3. Graph Inequalities
4. Find the Distance on a Real Number Line
5. Use the Laws of Exponents
6. Evaluate Square Roots

Real Numbers

Real numbers are numbers that can be written in decimal notation.

The set of real numbers includes:

- integers, positive and negative

  $\ldots -2, -1, 0, 1, 2 \ldots$

- rational numbers (numbers that can be written as fractions)

  $\frac{1}{2}, -\frac{2}{3}, .263$ (since $.263 = \frac{263}{1000}$)

- irrational numbers (numbers whose decimal representation neither repeats nor terminates)

  $\pi, \sqrt{2}, \sqrt{7}$

Evaluating an Algebraic Expression

To evaluate an algebraic expression, substitute for each variable its numerical value.

Example: Evaluate each expression if $x = 2$ and $y = -5$.

a. $x + 7y$

$x + 7y = 2 + 7(-5) = 2 + (-35) = -33$

b. $10xy$

$10xy = 10(2)(-5) = (20)(-5) = -100$

c. $\frac{-4x}{1-6y}$

$\frac{-4x}{1-6y} = \frac{-4(2)}{1-6(-5)} = \frac{-8}{1-(-30)} = \frac{-8}{1+30} = \frac{-8}{31}$

Finding the Domain of a Variable

Domain of a variable: The set of values that a variable in an expression may assume.

Example: Find the domain of $\frac{x}{x-9}$

Domain: All numbers except 9, which will make the denominator equal to 0.

Domain: $\{x \mid x \neq 9\}$ "The set of all x such that x is not equal to 9"

set-builder notation

Example: Find the domain of $\sqrt{x}$

Domain: All numbers greater than or equal to 0. (You can't take the square root of a negative number)

Domain: $\{x \mid x \geq 0\}$ "The set of all x such that x is greater than or equal to 0."

$$a^n \xleftarrow{\text{exponent}}$$

base

Laws of Exponents

$a^n = \overbrace{a \cdot a \cdot \cdots \cdot a}^{n \text{ factors}}$

$2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$ $\qquad -2^4 = (-1) \cdot 2 \cdot 2 \cdot 2 \cdot 2 = -16$ $\qquad (-2)^4 = (-2)(-2)(-2)(-2) = 16$

$a^0 = 1$

$5^0 = 1$ $\qquad (-3)^0 = 1$ $\qquad -3^0 = (-1)(1) = -1$

$x^0 = 1$

$a^{-n} = \dfrac{1}{a^n}$

$5^{-2} = \dfrac{1}{5^2} = \dfrac{1}{25}$ $\qquad x^{-2} = \dfrac{1}{x^2}$

$a^m a^n = a^{m+n}$

$5^2\, 5^4 = 5^{2+4} = 5^6 = 15{,}625$ $\qquad x^3 \cdot x^4 = x^{3+4} = x^7$

Note that the bases must be the same

$\dfrac{a^m}{a^n} = a^{m-n}$

$\dfrac{3^7}{3^4} = 3^{7-4} = 3^3 = 27$ $\qquad \dfrac{x^{10}}{x^6} = x^{10-6} = x^4$

$\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

$\left(\dfrac{7}{4}\right)^2 = \dfrac{7^2}{4^2} = \dfrac{49}{16}$ $\qquad \left(\dfrac{x}{y}\right)^4 = \dfrac{x^4}{y^4}$

$(a^m)^n = a^{mn}$

$(5^3)^2 = 5^6 = 15625$ $\qquad$ $(x^3)^4 = x^{3\cdot 4} = x^{12}$

Writing Expressions so that all Exponents are Positive

Example: $(4y^5)^{-2}$

$$(4y^5)^{-2} = \frac{1}{(4y^5)^2} = \frac{1}{(4^1y^5)^2} = \frac{1}{4^2y^{10}} = \frac{1}{16y^{10}}$$

$4 = 4^1$

power rule: multiply exponents

Example: $(x^{-1}y)^3$

$$(x^{-1}y)^3 = x^{-3}y^3 = \frac{y^3}{x^3}$$

power rule

Example: $\dfrac{4x^{-2}(yz)^{-1}}{2^3x^4y}$

$$\frac{4x^{-2}(yz)^{-1}}{2^3x^4y} = \frac{4}{2^3x^2(yz)^1y} = \frac{4}{8x^2\cdot y\cdot z\cdot y} = \frac{4}{8x^2yz}$$

Anything with a negative exponent in the numerator goes to the denominator with a positive exponent.

Example: $\left(\dfrac{5x^{-2}}{6y^{-2}}\right)^{-3}$

$$\left(\frac{5x^{-2}}{6y^{-2}}\right)^{-3} = \left(\frac{6y^{-2}}{5x^{-2}}\right)^3 = \left(\frac{6x^2}{5y^2}\right)^3 = \frac{6^3x^6}{5^3y^6} = \frac{216x^6}{125y^6}$$

Take reciprocal + write with a positive exponent.

power rule

Absolute Values

The absolute value of a real number a, denoted by the symbol $|a|$, is defined by the rules

$$|a| = a \text{ if } a \geq 0 \quad \text{and} \quad |a| = -a \text{ if } a < 0$$

Example: $|5|$ $= 5$

Example: $|-25|$ $= 25$

Evaluating Square Roots

In general, if a is a nonnegative square number, the nonnegative number b such that $b^2 = a$ is the principal square root of a and is denoted by $b = \sqrt{a}$.

Examples:

$\sqrt{25}$ $= 5$

$\sqrt{\frac{1}{49}}$ $= \frac{1}{7}$

$(\sqrt{6})^2$ $= 6$

$\sqrt{(-12)^2}$ $= 12$ (principal square root must be positive.)

Note: Negative numbers do not have square roots in the real number system.