

## Section A-3 Polynomial and Rational Expressions

### Objectives

1. Recognize Special Products
2. Factor Polynomials
3. Simplify Rational Expressions

Monomial: The product of a number and a variable raised to a nonnegative integer power. Thus a monomial is of the form

$$ax^k$$

where  $a$  is called the *coefficient* and  $k$  is called the *degree*.

Examples:

Monomial	Coefficient	Degree
$5x^2$	5	2
$-4x^5$	-4	5
$x^9$	1	9
10	10	0

Like terms—Two monomials having the same degree and the same variable.

$$\begin{array}{c} 2x^4, 4x^4 \\ \uparrow \nearrow \\ \text{Like Terms} \end{array}$$

$$\begin{array}{c} 6x^5, -3x^5, x^5 \\ \uparrow \nearrow \\ \text{Like terms} \end{array}$$

We can add and subtract monomials that are like terms.

Example:  $2x^2 + 4x^2 = 6x^2$

Binomial: The sum or difference of two monomials having different degrees.

Trinomial: The sum or difference of three monomials having different degrees.

Examples: Binomials

$$x^2 - 4$$

$$2x^5 + 10$$

Trinomials

$$x^3 - 2x + 1$$

$$4x^9 - x^2 + 2$$

A polynomial in one variable is an algebraic expression of the form:

$$a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

where  $a_n, a_{n-1}, \dots, a_1, a_0$  are the coefficients of the polynomial,  $n \geq 0$  is an integer, an  $x$  is a variable.

$a_n$  is the leading coefficient and  $n$  is the degree.

Examples:

Polynomial	Coefficients	Degree
$3x^2 + 2x + 1$	3, 2, 1	2
$4x^3 - 1 = 4x^3 + 0x^2 + 0x - 1$	4, 0, 0, -1	3

### Adding and Subtracting Polynomials

Example: Add  $2x^4 + 5x^3$  and  $4x^3 - 10x^2$ .

Combine like terms

$$2x^4 + \underline{5x^3} + \underline{4x^3} - 10x^2 = 2x^4 + 9x^3 - 10x^2$$

Example:  $(3y^5 + 12y^3 + 6) + (9y^4 + 10y^3 - 7)$

Combine like terms

$$3y^5 + \underline{12y^3} + \underline{6} + 9y^4 + \underline{10y^3} - \underline{7} = 3y^5 + 9y^4 + 22y^3 - 1$$

Example:  $(13y^5 + 6y^3 + 3) - (14y^4 - 10y^3 - 7)$

$$(13y^5 + 6y^3 + 3) - (14y^4 - 10y^3 - 7) \quad \text{distribute the minus sign}$$

$$= 13y^5 + \underline{6y^3} + \underline{3} - 14y^4 + \underline{10y^3} + \underline{7}$$

$$= 13y^5 - 14y^4 + 16y^3 + 10$$

combine like terms

Example:  $(7x^4 - 6) - (7x^7 + 5x^4 + 17)$

$$(7x^4 - 6) - (7x^7 + 5x^4 + 17)$$

distribute the minus sign

$$= \underline{7x^4} - \underline{6} - 7x^7 - \underline{5x^4} - \underline{17}$$

$$= -7x^7 + 2x^4 - 23$$

combine like terms

## Multiplying Polynomials

Example: Multiply  $2(x^2 + 6x - 1)$ .

$$2(x^2 + 6x - 1)$$

distribute the 2

$$= 2x^2 + 12x - 2$$

Example: Multiply  $5y(y^2 - 6y)$ .

$$\begin{aligned} & 5y(y^2 - 6y) && \text{distribute the } 5y \\ & = 5y^3 - 30y^2 \end{aligned}$$

Example: Multiply  $(x + 2)(x + 3)$ .

$$\begin{aligned} & (x + 2)(x + 3) \\ & = x^2 + 3x + 2x + 6 && \text{FOIL} \\ & = x^2 + 5x + 6 && \text{simplify} \end{aligned}$$

Example: Multiply  $(2x^2 - 7)(4x + 1)$ .

$$\begin{aligned} & 2(x^2 - 7)(4x + 1) && \text{Distribute the } 2 \\ & = (2x^2 - 14)(4x + 1) \\ & = 8x^3 + 2x^2 - 56x - 14 && \text{FOIL} \end{aligned}$$

Example: Multiply  $(5x - 7)(x^2 + 3x - 1)$ .

$$\begin{aligned} & (5x - 7)(x^2 + 3x - 1) && \text{Distribute the } 5x, \text{ then} \\ & && \text{distribute the } -7. \\ & = 5x^3 + 15x^2 - 5x - 7x^2 - 21x + 7 \\ & = 5x^3 + 8x^2 - 26x + 7 && \text{Simplify by combining like} \\ & && \text{terms.} \end{aligned}$$

## Factoring

What does it mean to factor a number? *Write as the product of factors.*

$$10 = 2 \cdot 5$$

$$12 = 2 \cdot 2 \cdot 3 \quad \text{or} \quad 4 \cdot 3$$

$$20 = 2 \cdot 2 \cdot 5 \quad \text{or} \quad 2 \cdot 10 \quad \text{or} \quad 4 \cdot 5 \quad \text{etc.}$$

## Factoring Polynomials

- Factor out the greatest common factor (GCF)
- Factor trinomials
- Factor a difference of squares
- Factor by grouping

### Factoring Polynomials (GCF)

Example: Factor  $6x + 2$ .

$$6x + 2 = 2(3x + 1)$$

↑  
GCF

Example: Factor  $3x^2 + 9x$ .

$$3x^2 + 9x = 3x(x + 3)$$

↑  
GCF



## Factoring by Grouping

Example:  $x^3 + 3x^2 + 2x + 6$ .

$$\begin{aligned} & \underline{x^3 + 3x^2} + \underline{2x + 6} \\ = & x^2(x+3) + 2(x+3) \\ = & (x+3)(x^2+2) \end{aligned}$$

Group first two terms together and last two terms together, then factor out GCF from each group.

Factor out GCF  $x+3$

Example: Factor  $x^3 + 2x^2 - x - 2$ .

$$\begin{aligned} & \underline{x^3 + 2x^2} - \underline{x - 2} \\ = & x^2(x+2) - 1(x+2) \\ = & (x+2)(x^2-1) \\ = & (x+2)(x+1)(x-1) \end{aligned}$$

$x^2-1$  is a difference of squares  
(see below)

Example: Factor  $x^3 + 3x^2 + x + 3$ .

$$\begin{aligned} & \underline{x^3 + 3x^2} + \underline{x + 3} \\ = & x^2(x+3) + 1(x+3) \\ = & (x+3)(x^2+1) \end{aligned}$$

## Factoring Polynomials: Trinomials

Example: Factor  $x^2 + 7x + 12$ .

$$\begin{aligned} & x^2 + 7x + 12 \\ = & (x+3)(x+4) \end{aligned}$$

Find two numbers that multiply to 12 and add to 7.

Example: Factor  $x^2 + 2x - 8$ .

$$\begin{aligned} & x^2 + 2x - 8 \\ = & (x+4)(x-2) \end{aligned}$$

Find two numbers that multiply to  $-8$  and add to 2.

Example: Factor  $3x^2 + 6x - 9$ .

Always try to factor out the GCF first.

$$\begin{aligned} & 3x^2 + 6x - 9 \\ = & 3(x^2 + 2x - 3) \\ = & 3(x+3)(x-1) \end{aligned}$$

Find two numbers that multiply to  $-3$  and add to 2. 3 and  $-1$

Example: Factor  $6x^2 + 11x - 7$ .

$$\begin{aligned} & 6x^2 + 11x - 7 \\ = & \underline{6x^2 + 14x} - \underline{3x - 7} \\ = & 2x(3x+7) - 1(3x+7) \\ = & (3x+7)(2x-1) \end{aligned}$$

When the coefficient of  $x^2$  isn't 1, use guess and check or the method below to factor.

Find two numbers that multiply to  $ac = -42$  and add to  $b = 11$

Numbers: 14,  $-3$

Break apart middle term.

Factor by grouping.

Example: Factor  $3x^2 - 10x - 25$ .

$$\begin{aligned} & 3x^2 - 10x - 25 \\ & \quad \swarrow \quad \searrow \\ = & \underline{3x^2 - 15x} + \underline{5x - 25} \\ = & 3x(x-5) + 5(x-5) \\ = & (x-5)(3x+5) \end{aligned}$$

Find two numbers that multiply to  $ac = -75$   
and add to  $b = -10$

Numbers:  $-15, 5$

Break apart middle term using the two numbers.

Factor by grouping.

### Factoring a Difference of Squares

$$x^2 - y^2 = (x + y)(x - y)$$

Example:  $x^2 - 25$

$$x^2 - 25 = (x + 5)(x - 5)$$

Example:  $9 - 16y^2$

$$9 - 16y^2 = (3 + 4y)(3 - 4y)$$

Example:  $x^4 - 25$ .

$$x^4 - 25 = (x^2)^2 - 5^2 = (x^2 + 5)(x^2 - 5)$$