Section A-5
Solving Equations

Objectives
1. Solve Linear Equations
2. Solve Quadratic Equations by Factoring
3. Solve Quadratic Equations by Using the Square Root Method
4. Solve Quadratic Equations by Using the Quadratic Formula

A linear equation in one variable is equivalent to the form

$$ax + b = 0$$

where a and b are real numbers and $a \neq 0$.

Solving Linear Equations

Example: Solve $x + 4 = 10$

$x + 4 = 10$

$x + 4 - 4 = 10 - 4$ subtract 4 from both sides

$x = 6$

Example: Solve $x - 10 = 30$

$x - 10 = 30$

$x - 10 + 10 = 30 + 10$

$x = 40$

Example: Solve $3x = -24$

$3x = -24$

$\frac{3x}{3} = \frac{-24}{3}$

$x = -8$

Example: Solve $3x + 18 = 12$

$$3x + 18 = 12$$

$$3x + 18 - 18 = 12 - 18$$

$$3x = -6$$

$$\frac{3x}{3} = \frac{-6}{3}$$

$$x = -2$$

Example: Solve $3 - 2x = 2 - 3x$

$$3 - 2x = 2 - 3x$$

add $2x$ to both sides

$$3 - 2x + 2x = 2 - 3x + 2x$$

$$3 = 2 - x$$

$$3 - 2 = 2 - x - 2$$

$$1 = -x$$

multiply both sides by -1

$$-1 = x$$

Example: Solve $5 - (2x - 1) = 10$

$$5 - (2x - 1) = 10$$

Distribute the minus sign across the parentheses.

$$5 - 2x + 1 = 10$$

$$6 - 2x = 10$$

$$6 - 2x - 6 = 10 - 6$$

$$-2x = 10$$

$$x = -5$$

Example: Solve $(x + 2)(x - 3) = (x - 3)^2$

$$(x+2)(x-3) = (x-3)(x-3)$$

FOIL

$$x^2 - 3x + 2x - 6 = x^2 - 3x - 3x + 9$$

Simplify each side

$$x^2 - x - 6 = x^2 - 6x + 9$$

Subtract x^2 from both sides.

$$-x - 6 = -6x + 9$$

$$-x - 6 + 6x = -6x + 9 + 6x$$

$$5x - 6 = 9 \longrightarrow 5x = 15 \rightarrow x = 3$$

Example: Solve $w(4 - w^2) = 8 - w^3$

$$w(4 - w^2) = 8 - w^3$$

Distribute the w

$$4w - w^3 = 8 - w^3$$

$$4w - w^3 + w^3 = 8 - w^3 + w^3$$

$$4w = 8$$

$$w = 2$$

Solving Quadratic Equations by Factoring

A quadratic equation is an equation equivalent to one of the form

$$ax^2 + bx + c = 0$$

where a, b, and c are real numbers and $a \neq 0$.

Example: Solve $x^3 - x^2 = 0$

$x^3 - x^2 = 0$ Factor out GCF

$x^2(x-1) = 0$ Set each factor equal to 0.

$x^2 = 0$ $\quad$ $x - 1 = 0$

$x = 0$ $\quad$ $x = 1$

Example: Solve $4z^3 - 8z^2 = 0$

$4z^3 - 8z^2 = 0$ Factor out GCF

$4z^2(z-2) = 0$ Set each factor equal to 0.

$4z^2 = 0$ $\quad$ $z - 2 = 0$

$z = 0$ or $z = 2$

Example: Solve $v^2 + 7v + 12 = 0$

$v^2 + 7v + 12 = 0$ Factor the trinomial

$(v+3)(v+4) = 0$ Set each factor equal to 0

$v + 3 = 0$ $\quad$ $v + 4 = 0$

$v = -3$ or $v = -4$

Example: Solve $2x^2 + 5x + 2 = 0$

$2x^2 + 5x + 2 = 0$ Factor the trinomial

$(2x+1)(x+2) = 0$ Set each factor equal to 0

$2x + 1 = 0$ $\quad$ $x + 2 = 0$

$-1 \quad -1$

$2x = -1$ $\quad$ $x = -2$

$x = -\frac{1}{2}$

Example: Solve $4x^2 + 9 = 12x$

$4x^2 + 9 = 12x$ — Pull everything to one side

$4x^2 - 12x + 9 = 0$ — Factor the trinomial

$(2x-3)(2x-3) = 0$ — Set each factor equal to 0. Since there are two identical factors, you need to do this only once.

$2x - 3 = 0$

$\frac{2x}{2} = \frac{3}{2}$

$x = \frac{3}{2}$

Example: Solve $x^3 + 4x^2 - x - 4 = 0$

$\underbrace{x^3+4x^2}\underbrace{-x-4} = 0$ — Factor by grouping

$x^2(x+4) - 1(x+4) = 0$

$(x+4)(x^2-1) = 0$ — Factor the difference of squares.

$(x+4)(x+1)(x-1) = 0$ — Set each factor equal to 0.

$x+4=0 \qquad x+1=0 \qquad x-1=0$

$x=-4 \qquad x=-1 \qquad x=1$

Example: Solve $3x^3 + 4x^2 = 27x + 36$

$3x^3 + 4x^2 = 27x + 36$ — Pull everything to the left side.

$\underbrace{3x^3+4x^2}\underbrace{-27x+36} = 0$ — Factor by grouping

$x^2(3x+4) - 9(3x+4) = 0$

$(3x+4)(x^2-9) = 0$ — Factor the difference of squares.

$(3x+4)(x+3)(x-3) = 0$ — Set each factor equal to 0.

$3x+4=0 \qquad x+3=0 \qquad x-3=0$

$-4\ \ -4$

$3x=-4 \qquad x=-3 \qquad x=3$

$x = -4/3$

Solving Quadratic Equations Using the Quadratic Formula

Solutions to an equation of the form $ax^2 + bx + c = 0$ are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example: Solve using the quadratic formula: $x^2 + 5x + 3 = 0$

$a = 1$
$b = 5$
$c = 3$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(1)(3)}}{2(1)}$$

$$x = \frac{-5 \pm \sqrt{25 - 12}}{2}$$

$$x = \frac{-5 \pm \sqrt{13}}{2}$$

Example: Solve using the quadratic formula: $2x^2 + 5x + 3 = 0$

$a = 2$
$b = 5$
$c = 3$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(2)(3)}}{2(2)}$$

$$x = \frac{-5 \pm \sqrt{25 - 24}}{4}$$

$$x = \frac{-5 \pm \sqrt{1}}{4}$$

$$x = \frac{-5 \pm 1}{4}$$

$$x = \frac{-5+1}{4} = -1 \quad \text{or} \quad x = \frac{-5-1}{4} = -\frac{6}{4} = -\frac{3}{2}$$

Solving Quadratic Equations Using the Square Root Method

Note: The square must be isolated first.

Example: Solve $x^2 = 36$

$$x^2 = 36$$
$$\sqrt{x^2} = \pm\sqrt{36}$$
$$x = \pm 6$$

Example: Solve $(x+2)^2 = 1$

$$(x+2)^2 = 1$$

Take square root of both sides

$$\sqrt{(x+2)^2} = \pm\sqrt{1}$$
$$x+2 = \pm 1$$

$x+2 = 1$ or $x+2 = -1$

$x = -1$ $\quad$ $x = -3$

Example: Solve $(3x-2)^2 = 4$

$$(3x-2)^2 = 4$$

Take square root of both sides

$$\sqrt{(3x-2)^2} = \pm\sqrt{4}$$
$$3x-2 = \pm 2$$

$3x-2 = 2$ or $3x-2 = -2$

$3x = 4$ $\quad$ $3x = 0$

$x = 4/3$ $\quad$ $x = 0$

Example: Solve $(5x+7)^2 + 4 = 29$

$$(5x+7)^2 + 4 = 29$$
$$-4 \quad -4$$

Isolate the square.

$$(5x+7)^2 = 25$$

Take the square root of both sides.

$$\sqrt{(5x+7)^2} = \pm\sqrt{25}$$
$$5x+7 = \pm 5$$

$5x+7 = 5$ $\quad$ $5x+7 = -5$

$5x = -2$ $\quad$ $5x = -12$

$x = \frac{-2}{5}$ or $x = \frac{-12}{5}$

Solving Absolute Value Equations

If a is a positive real number and if u is any algebraic expression, then

$$|u| = a \text{ is equivalent to } u = a \text{ or } u = -a.$$

Example: Solve the equation $|x+2| = 6$.

① Isolate the absolute value. (It's already isolated in this case)

② Write as two equations (without absolute value symbols) and solve.

$$\begin{array}{rcl} x+2 &=& 6 \\ -2 && -2 \\ \hline x &=& 4 \end{array} \qquad \begin{array}{rcl} x+2 &=& -6 \\ -2 && -2 \\ \hline x &=& -8 \end{array}$$

Example: Solve the equation $5|x-4| = 10$.

① Isolate the absolute value.

$5|x-4| = 10$ divide both sides by 5

$|x-4| = 2$

② Write as two equations

$$\begin{array}{rcl} x-4 &=& 2 \\ +4 && +4 \\ \hline x &=& 6 \end{array} \qquad \begin{array}{rcl} x-4 &=& -2 \\ +4 && +4 \\ \hline x &=& 2 \end{array}$$

Example: Solve the equation $|-3|x = 15$.

Here the variable is not in absolute values.

$|-3|x = 15$ $\quad |-3| = 3$

$3x = 15$ $\quad$ Divide both sides by 5.

$x = 5$

Example: Solve the equation $|2x-4| = -12$.

Trick question! Absolute values cannot be equal to negative numbers.