Section A.8
Interval Notation; Solving Inequalities

Objectives

1. Use Interval Notation
2. Use Properties of Inequalities
3. Solve Linear Inequalities
4. Solve Combined Inequalities
5. Solve Absolute Value Inequalities

Recall:

$a < x < b$ means that x is between a and b.

Example: $2 < x < 5$ means that x is between 2 and 5.
We can represent this as an <u>open interval:</u> (2,5) and graph the interval on a number line.

The endpoints 2 and 5 are not included in the interval. (<)

Example: $4 \leq x \leq 8$ means that x is between 4 and 8 including the endpoints 4 and 8.
We can represent this as a <u>closed interval</u>: [4, 8] and graph the interval on a number line.

The endpoints 4 and 8 are included in the interval (≤)

Example: $-2 \leq x < 7$ means that x is between -2 and 7, including -2.
We can represent this as a <u>half-open or half-closed interval</u>: [-2, 7] and graph the interval on a number line.

-2 is included in the interval; 7 is not.

Example: $x \leq 4$ means that x is less than or equal to 4. In interval notation, this is represented as $(-\infty, 4]$.

Example: $x > -3$ means that x is greater than to -3. In interval notation, this is represented as $(-3, \infty)$.

Solve Linear Inequalities

Examples of linear inequalities:

$3x - 5 < 10$

Infinite number of solutions:

$x = 1$, since $3(1) - 5 = -2$ which is less than 10

$x = 2$, since $3(2) - 5 = 1$ which is less than 10

⋮

The solutions to this inequality can be written as an interval.

To solve linear equalities, try to isolate the variable on one side. **Remember, if you multiply or divide both sides by a negative number, the inequality changes direction.**

Example: Solve the inequality $2x + 5 > 1$ and graph the solution set.

$2x + 5 > 1$

$2x + 5 - 5 > 1 - 5$ Subtract 5 from both sides

$2x > -4$

$\frac{2x}{2} > \frac{-4}{2}$ Divide both sides by 2.

$x > -2$

Solution set: $\{x \mid x > -2\}$

Interval notation: $(-2, \infty)$

Example: Solve the inequality $-2(x+3) < 8$ and graph the solution set.

$-2(x+3) < 8$

$\frac{-2(x+3)}{-2} < \frac{8}{-2}$ Divide both sides by -2. The inequality changes direction.

$x+3 > -4$

$x+3-3 > -4-3$

$x > -7$

Solution set: $\{x \mid x > -7\}$

Interval notation: $(-7, \infty)$

Example: Solve the inequality $-3x + 4 > \frac{1}{3}(x-2)$ and graph the solution set.

$-3x+4 > \frac{1}{3}(x-2)$

$3(-3x+4) > 3 \cdot \frac{1}{3}(x-2)$ Multiply each side by 3 to eliminate the fraction 1/3.

$-9x+12 > x-2$ Distribute

$-9x+12+2 > x-2+2$

$-9x+14 > x$

$-9x+14+9x > x+9x$

$\frac{14}{10} > \frac{10x}{10} \rightarrow \frac{7}{5} > x$

Solution set: $\{x \mid x < \frac{7}{5}\}$

Interval notation: $(-\infty, \frac{7}{5})$

Solving Combined Inequalities

Example: Solve $4 \leq 2x+2 \leq 10$ and graph the solution set.

$4 \leq 2x+2 \leq 10$

$4-2 \leq 2x+2-2 \leq 10-2$ Subtract 2 from each part

$2 \leq 2x \leq 8$

$\frac{2}{2} \leq \frac{2x}{2} \leq \frac{8}{2}$ Divide each part by 2

$1 \leq x \leq 4$

Solution Set: $\{x \mid 1 \leq x \leq 4\}$

Interval Notation: $[1, 4]$

Example: Solve $\frac{1}{3} < \frac{x+1}{2} \leq \frac{2}{3}$ and graph the solution set.

$\frac{1}{3} < \frac{x+1}{2} \leq \frac{2}{3}$

$6 \cdot \frac{1}{3} < 6 \cdot \frac{x+1}{2} \leq 6 \cdot \frac{2}{3}$ Multiply each side by the LCD, 6.

$2 < 3(x+1) \leq 4$

$\frac{2}{3} < \frac{3(x+1)}{3} \leq \frac{4}{3}$

$\frac{2}{3} < x+1 \leq \frac{4}{3}$

$\frac{2}{3} - 1 < x + 1 - 1 \leq \frac{4}{3} - 1$

$-\frac{1}{3} < x \leq \frac{1}{3}$

Solution Set: $\{x \mid -\frac{1}{3} < x \leq \frac{1}{3}\}$

Interval Notation: $(-\frac{1}{3}, \frac{1}{3}]$

Solving an Inequality Involving Absolute Values

"Less Than"

$|x| < a$ is equivalent to $-a < x < a$.
$|x| \leq a$ is equivalent to $-a \leq x \leq a$.

Example: Solve the inequality: $|x| < 3$.

$|x| < 3$ is equivalent to $-3 < x < 3$

Example: Solve the inequality $|2x| < 8$ and graph the solution set.

$|2x| < 8$ is equivalent to $-8 < 2x < 8$

$\frac{-8}{2} < \frac{2x}{2} < \frac{8}{2}$ Divide each part by 2.

$-4 < x < 4$

Solution Set: $\{x \mid -4 < x < 4\}$

Interval notation: $(-4, 4)$

Example: Solve the inequality $|x + 1| < 4$ and graph the solution set.

$|x+1| < 4$ is equivalent to $-4 < x + 1 < 4$

$-4 - 1 < x + 1 - 1 < 4 - 1$ Subtract 1 from each part

$-5 < x < 3$

Solution set: $\{x \mid -5 < x < 3\}$

Interval notation: $(-5, 3)$

Example: Solve the inequality $|x + 4| + 3 \leq 5$ and graph the solution set.

$|x+4| + 3 \leq 5$

Isolate the abs. value $|x+4| + 3 - 3 \leq 5 - 3$

$|x+4| \leq 2$ is equivalent to $-2 \leq x + 4 \leq 2$

$-2 - 4 \leq x + 4 - 4 \leq 2 - 4$ Subtract 4 from each part

$-6 \leq x \leq -2$

Solution set: $\{x \mid -6 \leq x \leq -2\}$

Interval Notation: $[-6, -2]$

Example: Solve the inequality $|x-4| < -3$ and graph the solution set.

This expression is positive for all values of x. There is no value of x that would make $|x-4|$ less than -3. No solution

"Greater Than"

$|x| > a$ is equivalent to $x < -a$ *or* $x > a$
$|x| \geq a$ is equivalent to $x \leq -a$ *or* $x \geq a$

Example: Solve the inequality $|x| > 3$.

$|x| > 3$ is equivalent to $x < -3$ or $x > 3$

Solution set: $\{x \mid x < -3 \text{ or } x > 3\}$

Interval notation: $(-\infty, -3)$ or $(3, \infty)$

Example: Solve the inequality $|x-4| \geq 3$ and graph the solution set.

$|x-4| \geq 3$ is equivalent to $x-4 \leq -3$ or $x-4 \geq 3$

$x-4+4 \leq -3+4$ or $x-4+4 \geq 3+4$

$x \leq 1$ or $x \geq 7$

Solution set: $\{x \mid x \leq 1 \text{ or } x \geq 7\}$

Interval notation: $(-\infty, 1]$ or $[7, \infty)$

Example: Solve the inequality $|x+4| - 2 > 6$ and graph the solution set.

$|x+4| - 2 > 6$

Isolate the abs. value: $|x+4| - 2 + 2 > 6 + 2$

$|x+4| > 8$ is equivalent to $x+4 < -8$ or $x+4 > 8$

$x+4-4 < -8-4$ or $x+4-4 > 8-4$

$x < -12$ or $x > 4$

Solution Set: $\{x \mid x < -12 \text{ or } x > 4\}$

Interval Notation: $(-\infty, -12)$ or $(4, \infty)$