Chapter 4: Probability

4-2 Fundamentals

What is probability? The chance or likelihood that an event will occur.

Probabilities are always between 0 and 1, and are represented as decimals or percentages.

P(certain event) = 1 or 100%
P(impossible event) = 0 or 0 %

Notation:

- P denotes a probability.
- A, B, and C denote specific events.
- P(A) denotes the probability of event A occurring.

There are three approaches to probability:

1. Subjective
2. Relative Frequency
3. Classical

<u>Subjective probability</u> – found by simply guessing or estimating

Examples: What is the probability that the Patriots will win the Super Bowl in 2010?
Let A = the event that the Patriots win the Super Bowl in 2010
0.80 or 80% (my opinion)

What is the probability that I will get a raise this year?
Let A = the event that I get a raise this year
0.10 or 10% (my opinion)

<u>Relative Frequency Approximation of Probability</u>--Conduct or observe a procedure a large number of times, and count the number of times the event A actually occurs. Based on these actual results, P(A) is estimated to be:

P(A) = number of times A occurred/number of times trial was repeated

Example: Out of 2000 flights from Boston to Washington in 2002, 179 were late.

If we select one flight at random, what is the probability that it is late?

179/2000 = 0.0895

Classical Approach--Perform a procedure that produces outcomes. (Rolling a die, flipping a coin, etc.) and ask a question about probability.

P(A) = (number of ways A can occur)/(Total number of different possible outcomes)

Example: Toss a coin three times. What is the probability that I toss three heads in a row?

List out all of the possible outcomes of the procedure:

HHH, HHT, HTH, THH, TTH, THT, HTT, TTT

This is called the sample space.

There are 8 equally likely outcomes. The probability of any one outcome is 1/8.

P(HHH) = 1/8
P(TTT) = 1/8
P(2 heads) = 3/8
P(2 tails) = 3/8

Example: What is the probability that if I randomly select someone, that their birthday will be in May?

There are 31 days in May
There are 365 days in the year
31/365 = 0.0849

Law of Large Numbers—As a procedure is repeated again and again, the relative frequency probability of an event tends to approach the actual probability.

Rare event rule—if the probability that an event occurs is less than 5%, the event is considered rare or unusual.
Example: Is it "unusual" to get a 12 when a pair of dice is rolled?
Let C = get a 12
List all of the possible outcomes.

{(1,1) (1,2)…….(6,5) (6,6)}
P(C) = 1/36
Yes

The complement of event A, denoted by $\overline{A}$, consists of all outcomes in which event A does not occur.

Example: if A = the event of drawing a black card from a deck
$\overline{A}$ = the event of not drawing a black card from a deck = drawing a red card

4-3 Addition Rule

Simple event-an outcome or an event that cannot be further broken down into simpler components.

Compound event-an event combining two or more simple events.

Up until now, we've been talking about probabilities of simple events.

Example: Suppose that 30 prospective employees were subjected to a drug test.

	Positive Drug Test	Negative Drug Test	Total
Women	3	12	15
Men	5	10	15
Total	8	22	30

Simple event: If one of the 30 prospective employees is selected, what is P(positive drug test)?

Simple event: If one of the 30 prospective employees is selected, what is P(woman)?

Compound event: If one of the 30 prospective employees is selected, what is P(positive drug test **or** woman)?

Notation for the Addition Rule: P(A or B) = P(A occurs *or* B occurs *or* they both occur)

Addition Rule for Probability: **P(A or B) = P(A) + P(B) – P(A and B)**

Example: If one of the 30 prospective employees is selected, what is P(positive drug test or woman)?

Let A = the event of a positive drug test
Let B = the event of selecting a woman

P(A or B) = P(A) + P(B) – P(A and B) = 8/30 + 15/30 – 3/30 = 23/30

Mutually Exclusive Events: If P(A and B) = 0, the events cannot occur together. They are disjoint or mutually exclusive.

Addition Property for Mutually Exclusive Events: **P(A or B) = P(A) + P(B)**

Example: If an individual is selected, find P(Type A or Type B blood).

Blood Type	Percentage
O	.46
A	.40
B	.10
AB	.04

P(A or B) = 0.40 + 0.10 = 0.50 (mutually exclusive events)

Example: Roll a die
A = roll a 3
B = roll a 4

Events A and B are mutually exclusive.

P(A or B) = P(A) + P(B) = 1/6 + 1/6 = 2/6 = 1/3

Example: A = will rain this weekend
$\overline{A}$ = will not rain this weekend

If a meteorology estimates P(A) = .70, the P($\overline{A}$) = .30.

Note that P(A) + P($\overline{A}$) = 1

4-4 Multiplication Rule

Multiplication Rule—Used to find the probability that A occurs in a first trial and B occurs in a second trial.

Example: What is the probability that in two coin tosses, a head is tosses first and a tail is tossed second?

Example: What is the probability that if I roll a die and then toss a coin, a 4 is rolled and a tails is tossed?

P(A and B) = P(event A occurs in the first trial and B occurs in the second)

<u>Example:</u> Suppose we roll a die and flip a coin.

Let A = rolling a die and getting a 4
B = flipping a coin and getting tails

P(A) = 1/6
P(B) = 1/2

What is P(A and B)?

One way is to list the sample space:

1,H	3,H	5,H	1,T	3,T	5,T
2,H	4,H	6,H	2,H	4,H	6,H

There are twelve equally likely outcomes, so P(A and B) = 1/12

An alternative way is to use the formula

P(A and B) = P(A)* P(B) when A and B are independent

If events are <u>independent</u>, the occurrence of one doesn't affect the occurrence of the other.

For independent events,

If P(A) = 1/6 and P(B) = 1/2, then P(A and B) = 1/6*1/2 = 1/12

Example: if you take a 10 question T/F exam, what is the probability that you will guess and get all questions right?

(0.5)(0.5)***(0.5) = (0.5)^10 = 0.000977

<u>Example</u>: A bowl contains 3 yellow and 2 blue marbles. What is the probability that you choose a yellow marble on the first draw, replace it, and then choose a blue marble on the second draw.

A = yellow marble on first draw
B = blue marble on second

A and B are independent because we are sampling <u>with replacement.</u>

P(A and B) = 3/5 * 2/5

<u>Example:</u> What is the probability of getting a blue on the first and a blue on the second, when you sample with replacement?

2/5*2/5 = 4/25 = .16

If we sample without replacing the first marble, then A and B are <u>dependent.</u> The occurrence of one is related to the occurrence of the other.

If you have 3 yellow and 2 blue marbles,

P(A and B) = 3/5 * 2/4

For dependent events, **P(A and B) = P(A)*P(B|A)**, where P(B|A) represents the probability of event B occurring after it is assumed that event A has already occurred.

<u>Example:</u> What is the probability of getting a blue on the first and a blue on the second, when you sample without replacement?

2/5*1/4 = 2/20 = 1/10 = .1

If a sample size is no more than 5% of the size of the population, treat the selections as being independent (even if the selections are made without replacement, so they are technically dependent).

Example: In a batch of 8,000 clock radios 5% are defective. A sample of 13 clock radios is randomly selected without replacement from the 8,000 and tested. The entire batch will be accepted if every item in the sample is not defective. What is the probability that the entire batch will be accepted?

P(all good) = $(0.95)^{13}$ = .5133420833

4-5 Multiplication Rule: Complements and Conditional Probability

Probabilities of at least one

"at least one" = 1 or more

complement of "at least one = none

Example: Find the probability of a couple having at least 1 boy among 4 children.
Although we solved this problem previously by listing all of the outcomes, there is another way to solve it.

Let A = the event of getting at least one boy
Then $\bar{A}$ = the event of not getting at least one boy = no boys = all girls

P($\bar{A}$) = 1/2*1/2*1/2 *1/2= 1/16
P(A) = 1 – P($\bar{A}$) = 1 – 1/16 = 15/16

There is a 15/16 probability that if a couple has 4 children, at least 1 of them is a boy.

P(at least one) = 1 – P(none)

Example: A batch of 2500 CDs are manufactured and 2% are defective. 4 CDs are selected and tested, and the entire batch of CDs will be rejected if at least one CD is defective. What is the probability that the batch will be rejected?

A = at least one defective CD
$\bar{A}$= no defective CDs

P( ) = $(0.98)^4$ = .92236816
P(A) = 1 - .92236816 = .07763184

Conditional probability P(B|A) – "The probability of B, given that A has already occurred"-- assume that A has occurred, and calculate the probability that event B will occur.

P(B|A) = P(A and B)/P(A)

Example: Refer to the following table regarding Titanic mortality.

	Men	Women	Boys	Girls	Total
Survived	332	318	29	27	706
Died	1360	104	35	18	1517
Total	1692	422	64	45	2223

Question: If we randomly select someone who died, what is the probability of getting a man?

Answer:
Count the number of people who died: 1517
Of that subgroup, how many were men?
So the probability is 1360/1517 = 0.897

Alternatively, using the formula, where B = man, A = died

P(B|A) = P(B and A)/P(A)
P(B and A) = 1360/2223
P(A) = 1517/2223
P(B|A) = P(B and A)/P(A)= 1360/1517 = 0.897

Question: If we randomly select someone who survived, what is the probability of getting a woman?

Answer:
Count the number of people who survived: 706
Of that subgroup, how many were women? 318
So the probability is 318/706 = 0.450

Alternatively, using the formula, where B = woman, A = survived

P(B|A) = P(B and A)/P(A)
P(B and A) = 318/2223
P(A) = 706/2223
P(B|A) = P(B and A)/P(A)= 318/706 = 0.450