

Chapter 5 Probability Distributions

5-2 Random Variables

A *random variable*, X , is a variable that has a single numerical value, determined by chance, for each outcome of a procedure.

examples: the number of heads in ten tosses of a coin
 the number of girls in a family with 4 children

A *probability distribution* is a graph, table, or formula that gives the probability for each value of the random variable (i.e. which values are more likely; which values are less likely).

examples: we would be most likely to get 5 heads; 0 or 10 heads would be very unusual

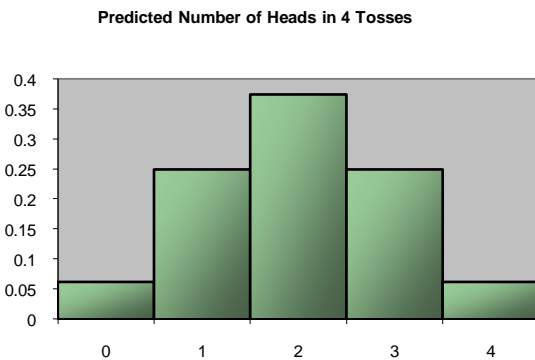
Probability distributions describe what will probably happen.

example: Let X = the number of heads in four tosses of a coin. The following is a probability distribution for X .

X	$P(X)$
0	0.0625
1	0.25
2	0.375
3	0.25
4	0.0625

$\sum P(x) = 1$ where x assumes all possible values
 $0 < P(x) < 1$ for every individual value of x

We could draw a probability histogram for the probability distribution.



Mean, Variance and Standard Deviation of a Probability Distribution

Mean = $\mu = \sum x \cdot P(x)$ This is also called “expected value.” This is what we would expect the average value of X to be if the experiment were repeated many times.

$$\text{Variance} = \sigma^2 = \sum [(x - \mu)^2 \cdot P(x)]$$

or

$$\text{Variance} = \sigma^2 = \sqrt{\sum [x^2 \cdot P(x)] - \mu^2}$$

$$\text{Standard Deviation} = \sigma = \sqrt{\sigma^2}$$

For the previous problem:

X	P(X)	X*P(X)	X - μ	(X - μ) ²	(X - μ) ² P(X)
0	0.0625	0	-2	4	0.25
1	0.25	0.25	-1	1	0.25
2	0.375	0.75	0	0	0
3	0.25	0.75	1	1	0.25
4	0.0625	0.25	2	4	0.25

$$\mu = \sum X \cdot P(X) = 2$$

$$\sigma^2 = \sum (X - \mu)^2 P(X) = 1$$

$$\mu = \sum x \cdot P(x) = 2$$

$$\sigma^2 = \sum [(x - \mu)^2 \cdot P(x)] = 1$$

Example: In a game, you have a 1/23 probability of winning \$62 and a 22/23 probability of losing \$10. What is your expected value?

$$\text{Expected value} = 62 \cdot (1/23) + (-10) \cdot (22/23) = -6.87$$

On average, you would lose about \$6.87 per game.

Example: Suppose you buy 1 ticket for \$1 out of a lottery of 1,000 tickets where the prize for the one winning ticket is to be \$500. What is your expected value?

$$\text{Expected Value} = 499 * (1/1000) + (-1) * (999/1000) = -0.5$$

Example: A 28-year-old man pays \$234 for a one-year life insurance policy with coverage of \$110,000. If the probability that he will live through the year is 0.9992, what is the expected value for the insurance policy?

$$\text{Expected value} = (109,766) * (0.0008) + (-234) * (0.9992) = -146$$

Round-off rule for $\mu, \sigma, \text{and } \sigma^2$

Round results by carrying one more decimal place than the number of decimal places used for the random variable x . If the values of x are integers, round $\mu, \sigma, \text{and } \sigma^2$ to one decimal place.

On the calculator: Enter the values for X in L1, and the values for $P(X)$ in L2. Then press STAT → CALC → 1-Var Stats → Enter → L1, L2 → Enter

\bar{X} = mean

σ_x = standard deviation

5-3 Binomial Probability Distributions

Allow us to deal with situations that result in two possible outcomes (boy/girl, for/against, heads/tails, right/wrong)

Characteristics

- a fixed number of observations, called trials, denoted by n
- the trials must be independent
- each trial must have all outcomes classified into two categories, labeled success or failure
- the probability of success (p) and failure (q) must remain constant for each trial
- x = the number of successes in n trials
- $P(x)$ = the probability of getting exactly x successes in n trials

Binomial probability formula: $P(x) = \frac{n!}{x!(n-x)!} \cdot p^x q^{(n-x)}$

n = number of trials

x = number of "successes"

p = probability of success

q = probability of failure = $1-p$

Example: It has been reported that 30% of all credit card users pay their bills in full each month. If we select 5 credit card holders at random, what is the probability that exactly one pays his/her bills every month?

success = paying the bills every month

$n = 5$

$p = P(\text{success}) = .3$

$q = P(\text{failure}) = .7$

$x = 1$

$$P(x) = \frac{n!}{x!(n-x)!} \cdot p^x q^{(n-x)}$$

$$P(1) = \frac{5!}{1!(5-1)!} (.3)^1 (.7)^4 = .36015$$

Example: Among companies doing highway or bridge constructions, 80% test employees for substance abuse. A study involves the random selection of 10 such companies. What is the probability that exactly 9 test for substance abuse?

$n = 10$
 $p = .80$
 $q = .20$
 $x = 9$

$$P(9) = \frac{10!}{9!(10-9)!} (.8)^9 (.2)^1 = .2415919104$$

What is the probability that 9 or more test for substance abuse?

9 or more means $x = 9$ or $x = 10$

$$P(10) = P(10) = \frac{10!}{10!(10-10)!} (.8)^{10} (.2)^0 = .1073741824$$

$$P(9 \text{ or more}) = P(9) + P(10) = .2415919104 + .1073741824 = .3489660928$$

To use the calculator, press 2nd VARS (DISTR) → binompdf → enter n, p, x → ENTER

Use binompdf to find the probability of exactly x successes.

Use binomcdf to find the probability of x or fewer successes.

Example (continued) What is the probability that 6 or fewer test for substance abuse (i.e. $X = 0, 1, 2, 3, 4, 5, 6$)

method 1: Use the binomial formula—very time-consuming

method 2: Use calculator: 2nd VARS (DISTR) → binomcdf → enter n, p, x → ENTER

$$\text{binomcdf}(10, .8, 6) = .1208738816$$

Example: What is the probability that more than 3 test for substance abuse? (i.e. $X = 4, 5, \dots, 10$)

The complement of $X > 3$ is $X \leq 3$ (i.e. $X = 0, 1, 2, 3$).

$$P(X > 3) = 1 - P(X \leq 3) = 1 - \text{binomcdf}(10, .8, 3) = .9991356416$$

Example: What is the probability that 7 or more test for substance abuse?

$$.P(X \geq 7) = 1 - P(X \leq 6) = 1 - \text{binomcdf}(10, .8, 6) = 8791261184$$

Example: What is the probability that 2 or fewer test for substance abuse?

$P(X \leq 2) = \text{binomcdf}(10, .8, 2) = .0000779264$ very unusual—indicates that companies are probability not testing at the 80% rate after all.

5-4 Mean , Variance, and Standard Deviation for the Binomial Distribution

Example: A student is unprepared for a surprise true/false test with 4 questions, and all of their answers are guesses. If x = the number of correct answers, x is a binomial random variable.

$$n = 4$$

$$p = .50$$

x = number of correct answers

X	P(x)
0	0.062
1	0.250
2	0.375
3	0.250
4	0.062

We could calculate the mean and variance of this binomial distribution by using the formulas

$$E(x) = \mu = \sum [x \cdot P(x)]$$
$$\sigma^2 = \sum [(x - \mu)^2 \cdot P(x)]$$

But...there is an easier way.

For the binomial distribution,

$$\mu = np$$

$$\sigma^2 = npq$$

$$\sigma = \sqrt{npq}$$

example: (continued) $n = 4$, $p = 0.5$, and $q = 0.5$

$$\mu = np = 4 \cdot (0.5) = 2$$

$$\sigma^2 = npq = 4 \cdot (0.5)(0.5) = 1$$

$$\sigma = \sqrt{\sigma^2} = 1$$

Example: In a certain town, 36% of voters favor a given ballot measure. For groups of 30 voters, find the mean, variance, and standard deviation for the number who favor the measure.

$$n = 30$$

$$p = 0.36$$

$$q = 0.64$$

$$\mu = np = (30)(0.36) = 10.8$$

$$\sigma^2 = npq = (30)(0.36)(0.64) = 6.912$$

$$\sigma = \sqrt{\sigma^2} = 2.629068276$$

Values are unusual if they lie outside of these limits:

$$\text{maximum usual value: } \mu + 2\sigma$$

$$\text{minimum usual value: } \mu - 2\sigma$$

example (continued)

$$\text{maximum usual value: } \mu + 2\sigma = 10.8 + 2(2.629068276) = 16.05813655$$

$$\text{minimum usual value: } \mu - 2\sigma = 10.8 - 2(2.629068276) = 5.541863448$$

In groups of 30 voters, we would expect that between 5.54 and 16.06 would support the ballot measure.

Example: A survey for brand recognition is done and it is determined that 68% of consumers have heard of Dell Computer Company. A survey of 800 randomly selected consumers is to be conducted. For such groups of 800, would it be unusual get 526 consumers who recognize the Dell Computer Company name?

$$n = 800$$

$$p = 0.68$$

$$q = 0.32$$

$$\mu = (800)(0.68) = 544$$

$$\sigma^2 = (800)(0.68)(0.32) = 174.08$$

$$\sigma = \sqrt{\sigma^2} = 13.193938$$

$$\text{maximum usual value: } \mu + 2\sigma = 570.387876$$

$$\text{minimum usual value: } \mu - 2\sigma = 517.612124$$

No, it would not be unusual.